CONCEPTUAL MODELS OF THE RESOURCE ALLOCATION DECISION PROCESS IN HIERARCHICAL DECENTRALIZED ORGANIZATIONS

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ERRATA

page 118, line 13 : "objectives if" should read "objectives is"

page 161, line 11 : "$\sigma_k \geq z_k^*(\alpha_k^1) - (\underline{\alpha}_k^1)'\underline{\pi}_k^2 + (\underline{\alpha}_k^2)'\underline{\pi}_k^1$ "

should read "$\sigma_k \geq z_k^*(\alpha_k^1) - (\underline{\alpha}_k^1)'\underline{\pi}_k^1 + (\underline{\alpha}_k^2)'\underline{\pi}_k^1$ "

page 161, line 12 : "$\sum_{k=1}^{n} P_k\underline{\alpha}_k \leq \underline{q}$" should read "$\sum_{k=1}^{n} P_k\underline{\alpha}_k^2 \leq \underline{q}$"

page 163, line 1 : should read

"$\sigma_k^t \geq z_k^*(\underline{\alpha}_k^j) - (\underline{\pi}_k^{*j})'\underline{\alpha}_k^j + (\underline{\pi}_k^{*j})'\underline{\alpha}_k^{t+1}$ (6-9) "

page 166, next to last line: "monotonically increase" should read "monotonically decrease."

page 168, line 3 : should read

"$\sigma^t \geq \sum_{k=1}^{n} [z_k^*(\alpha_k^j) - (\underline{\pi}_k^{*j})'\underline{\alpha}_k^j + (\underline{\pi}_k^{*j})'\underline{\alpha}_k^{t+1}$

page 175, line 17 : "given goal, $\underline{\alpha}_k^t$," should read "given goal $\underline{\alpha}_k^j$ "

page 176, line 2 : "$\underline{\alpha}_k^{j}{}'\underline{\pi}_k^{t+1}$ " should read "$\underline{\pi}_k^{j}{}'\underline{\alpha}_k^{t+1}$ "

page 176, line 8 : "the goals β_k" should read "the goals α_k "

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SUMMARY

The primary focus of this research study has been on the development, interpretation, and analysis of analytical models for conceptualizing the resource allocation decision process in hierarchical decentralized organizations. The three main classes of results are:

(1) The integration under a common framework (from the viewpoint of someone interested in structuring information flow in organizations) of existing mathematical procedures for characterizing the resource allocation process.

(2) Mathematical decomposition theory is used to develop a number of behavioral propositions regarding organizations.

(3) The development of negotiation models for coordinating resource allocation decisions when there exists conflict between levels over objectives.

The integrating framework recognizes that there can exist conflict between subordinate decision making units over limited resources and between levels over objectives. To overcome these conflicts there exists two main classes of coordination mechanisms: coordination through goal intervention and coordination through constraint intervention.

When goal and constraint intervention techniques are applied in an organization with no conflict between levels over objectives, they are often called pricing and resource budgeting mechanisms. The conditions under which each of these mechanisms work, their economic and

behavioral implications, and representative algorithms for describing the coordination procedure are given.

Some of the behavioral implications inherent in pricing and budgeting techniques are studied. Specifically, it is shown that:

(1) The only conflict is a result of the subordinates competing for limited organizational resources.

(2) The structure of the organization has no effect on the final solution reached.

(3) The subordinate decision making units have no autonomy.

(4) The allocation decision reflects only the objectives of the superordinate.

It is also shown that neither pricing mechanisms nor budgeting procedures are suitable for coordinating in an organization where there exists conflict between levels over objectives.

A set of coordination mechanisms called negotiation models which allow the resource allocation decision to be influenced by both the superordinate and subordinates is introduced. This model has its roots in some work by Ruefli, Kelley, and Benders. The important aspects of these models are:

(1) They allow for informational autonomy on the part of both the superordinate and the subordinates.

(2) The structure of the organization can affect the final decision. In other words, the final resource allocation program selected may be different under different organization structures. For example, the decisions made under a centralized structure may differ from those made under a decentralized structure.

(3) The goal setting behavior of the superordinate and the subordinates explicitly takes into account the "bounded rationality" of man, i. e., the decision makers concentrate only on certain aggregate measures of performance rather than trying to optimize some grandiose utility function.

(4) The models explicitly allow for both the superordinate and the subordinates to have their own set of goals.

(5) In the case of the goal partitioning procedure the iterative information exchange process between superordinate and subordinates converges in a finite number of information exchanges to a solution which minimizes the total weighted deviation from the goals set by the superordinate and the subordinates.

(6) The goal partitioning procedure may lead to a resource allocation plan being selected which is different from one which would be selected by the superordinate or the subordinates acting in isolation; however, it will lead to a program which reflects both the goals of the superordinate and the goals of the subordinates.

CHAPTER I

INTRODUCTION TO RESOURCE ALLOCATION DECISION MAKING IN HIERARCHICAL DECENTRALIZED ORGANIZATIONS

Management scientists have traditionally been concerned with the strategic problems of management [7,8, 116]. Ackoff [2, p. 21] states that management scientists were originally concerned with finding effective solutions to specific organizational problems, e. g., how to "optimally" schedule production, or where to "optimally" locate warehouses. But they found that too many of their solutions were not put into operation, and of those that were, too few survived the inclination of organizations to return to familiar ways of doing things. A reason suggested by some writers [2,6,84,116] is the absence of conceptual models and analysis of these models for organizational processes such as decision making, connunication, etc. For example, Messarovic, Macko and Takahara [84, p. 16] assert that, "at present there may be more researchers worrying about how to optimally adjust parameters in feedback control systems than there are researchers worrying about the quantitative aspects of control and communication processes in organizational-type systems." Apparently, there are several professionals (including the author) who believe that conceptual models of organizational processes can provide insights and knowledge about how the flow of information and the organizational structure can affect decision making. This research is directed toward that end. The purpose of this dissertation is to present and analyze mathematical models of one organizational process, resource allocation decision making. The study is aimed at a specific kind of environment, viz., a hierarchical decentralized organization. The results of the research would seem relevant to a challenge of the future made by Wagner [116, p. 1277]. Wagner suggested that the study and design of structures of management that are appropriate to the goals and environments of large scale organizations might lead to important laws underlying the science of decision making.

An important aspect of this research is its integrative nature. Professionals in at least four different technical areas have been concerned with conceptual models of the resource allocation process in hierarchical decentralized organizations. Historically, economists studied the resource allocation process in a decentralized setting first. Malinvaud [78, p. 179] presents a brief history of the literature on decentralized planning in economics and states that the main concern of economists has been in showing that under conditions of perfect competition a centralized planning agency could do no better (in terms of an overall utility function) than a number of entrepreneurs operating independently through markets. Thus, since economists assumed that the firms within the economy were small enough to be directed by the entrepreneur, and were within a purely competitive setting, there was no need to concern themselves with the structure or internal control mechanisms of the firm [126]. Organizational behavioralists have also addressed the problem of decentralized decision making. Their studies have mainly been based on extensive personal experiences and empirical experiments. Their results have been characterized by suggestions for more decentralized and participative decision making [75, 80] and often ignore the economic aspects of organizations [126, p. 51]. With some notable exceptions [31, 35, 107, 110] analytical models have not been developed to support these results and provide theoretical justification for their conclusions.

The literatures of operations research/management science and systems theory have also concerned themselves with decentralized structures. Operations researchers, e. g., [37, 72], have studied and developed procedures to decompose a large mathematical programming problem into subproblems, and then solve these smaller subproblems possibly many times in order to get a solution to the large problem. Often after the development of such a decomposition technique, someone will note that a decentralized economic interpretation can be associated with the procedure.[1] Systems theorists have concerned themselves with problems similar to those considered by operations researchers; however, the motivation for their studies has generally arisen from some hierarchical physical system. For example, Messarovic, Macko and Takahara [84, p. 13] have studied multilevel electric power systems. The result was a large mathematical problem which must be decomposed in a way that makes sense in terms of the physical systems.

Thus, economists, organizational behavioralists, operations researchers, and systems theorists have studied segments of the problem of resource allocation in a hierarchical decentralized structure. However, the organizational behavioralists have tended to ignore analytical

[1]For example, see Dantzig's footnote [36, p. 462].

models, while the others have either ignored or made restrictive assumptions about behavior. The author believes that this dissertation makes a significant contribution in integrating this previous work within a common framework.

Conceptual models which would provide insights and theoretical justification for many organizational processes have been virtually non-existent. Several well known professionals are now appealing for research in this area. Significant amounts of mathematical theory and behavioral study are available for constructing and analyzing these conceptual models. This research develops a mathematical framework for the study of resource allocation decision making within decentralized hierarchical organizations.

Decentralized Decision Making in a Hierarchical Organization

The purpose of this section is to define what is meant by resource allocation decision making, hierarchical structure, and decentralization. The advantages and disadvantages of decentralized decision making are also discussed.

For purposes of this research, an organization is viewed as a goal seeking system consisting of goal seeking subsystems[84]. As Simon has said, "large organizations are almost universally hierarchical in nature." [106, p. 40] A typical organization chart supports his statement. By hierarchical it is meant that the organization is composed of a vertical arrangement of subsystems where higher level subsystems have some priority of action or right of intervention over lower level subsystems, and where higher level subsystems must depend upon the performance of lower level subsystems [84, p. 34]. The existence of a hierarchical structure is not a characteristic that is peculiar to human systems. It is common to nearly all complex systems [84, 106].

The near universality of hierarchy in complex systems suggests that there is something fundamental in this structural principle. The reasons often cited for hierarchical structure in organizations are:

(1) Hierarchical structure allows for efficient accomplishment of the objectives of the organization in terms of time and cost, i.e., they allow for division of labor and specialization of function [106,126].

(2) Among systems of a given size and complexity, hierarchical systems require less information transmission between their parts than do non-hierarchical systems [106,126].

For these reasons Simon concludes that, "an organization will tend to assume a hierarchical form whenever the task environment is complex relative to the problem solving, communicating powers of the organization members and their tools. The organizations of the future, then, will be hierarchies, ..." [105, p. 43].

Resource allocation decision making within a hierarchical structure means that an organization which has a hierarchical structure faces the decision of how much of each resource, e. g., money, time, manpower, facilities, etc., should be alloted to each project or activity currently being considered by the organization. Thus, a hierarchical structure does not necessarily imply anything about where in the structure this allocation decision is made. However, it is important to consider the effects of the hierarchical structure on the amount and nature of information available to a decision maker.

Throughout this dissertation it is assumed that resource allocation decisions are made at specific identifiable points in the organization. This assumption is much like Connolly's concept of diffuse decision making [34]. Connolly describes a "diffuse" decision process as having four characteristics: (1) the decisions cover extended periods of time; (2) they are multi-person processes; (3) the participants are separated by non-trivial distances; and (4) the processes cover several organization levels. In this work the decision process can have characteristics (2) through (4). The final program of resource allocations are the culmination of decisions made at different points in the organization. It is assumed that the decision making process at a specific point in the organization can be modelled as the solution of a mathematical programming problem in which the constraints represent the technological and other restrictions imposed on the decision as perceived by the decision making unit. As with all mathematical programming problems, the objective function is a maximization or minimization operation, but this does not necessarily imply an unlimited search for the "best" solution. As will be shown later it can also represent satisficing, a concept usually attributed to Simon [105, p. XXV].

This research will also assume that resource allocation decisions are made at each level in the organization and by each unit within each level (these are the specific identifiable points mentioned earlier). This leads one to the term: resource allocation decision making in a decentralized hierarchical organization. It is virtually impossible to speak in absolute terms as to whether an organization is centralized or decentralized. As Simon states, "An administrative organization is centralized to the extent that decisions are made at relatively high levels in the organization; decentralized to the extent that discretion and authority to make important decisions are delegated by top management to lower levels of executive authority." [104, p. 1] Simon's statement implies that the organizational structure is hierarchical in nature. In fact, as Zannetos [126, p. 54] has noted, any organization short of utopian decentralization is hierarchical. However, the converse is not true, i. e., a hierarchical structure does not necessarily imply decentralized decision making. It is possible for a completely centralized organization to have a hierarchical structure for purposes of carrying out the decisions which are made by the central decision maker.

A totally centralized and a totally decentralized organization are myths, i. e., they do not exist in the real world. Instead most organizations fall somewhere between the two extremes of completely centralized and completely decentralized depending upon the amount of delegation of decision making authority [121]. One of the concerns of this research is the effect that certain coordination intervention mechanisms have on the centralization-decentralization properties of the organization.

Decentralized decision making in a hierarchy does offer some desirable attributes. The ones most commonly cited in the literature are:

(1) There is a saving in the amount of information which must be transmitted between decision making units [30, 60, 82, 105]. In a centralized structure the central decision making unit must be informed

of all alternatives and projects which exist throughout the organization; whereas a decentralized structure allows for more informational autonomy. In a decentralized setting, not all the data on alternatives and projects needs to be communicated. Instead, only certain summary information such as profit, cost, etc. is required. Thus, no individual decision unit in the organization need have complete information.

(2) A decentralized organization explicitly takes into account the cognitive inability of a large organization to solve their resource allocation problem because it is too complex [70, 108]. This is reflected by the overall complex problem being replaced by a sequence of smaller and less complex problems. These smaller less complex problems help to facilitate the decentralized assignment of the smaller problems to eliminate undue interunit dependencies [70, p. 545].

(3) A decentralized hierarchy may allow a feasible resource allocation program to be found rapidly [56, 82, 105]. This feature is related to not having to communicate information up to the central decision maker and then wait for a decision to be handed down. Sometimes the coordination of decision processes is iterative so that summary information is passed up and down the hierarchy several times, and the final decision may be delayed. The number of iterations may depend upon the specific decision to be made, e. g., crisis versus annual budgeting.

(4) Under a decentralized structure the operating rules of the system take the form of "do whatever is necessary to meet some objective" [9, p. 400]. Thus decentralization may improve the allocation of responsibility and allow top management to recognize more easily successes and

failures [106, p. 44].

(5) In a decentralized hierarchy the subordinate units are allowed greater participation in the decision making process. This participation may result in the subordinate decision making units being better motivated and have a deeper commitment to the final decision [18, 60]. For example, Tannebaum and Massarik [111] found that a decentralized structure for making resource allocation decisions increased morale and productivity.

Often, centralized decision making is impossible because of the exorbitant demands it places on the information system in a large, complex organization. Therefore, it is essential that any mathematical model of a decentralized organization allow the subordinates to have informational autonomy. This means that a mathematical model must not require the superordinate to have complete information. The other attributes such as increased motivation which can result from decentralization are important but not essential from a modelling viewpoint.

Although a hierarchical decentralized structure in an organization offers some desirable characteristics, it also creates coordination and control problems. A hierarchical structure tends to break the overall organization goals down into subgoals and distribute them over several levels and decision making units [35, p. 19]. The specialization of this decision making can create some conflicts within the organization. A major source of conflict is when limited resources such as capital, manpower, etc. must be distributed over several levels and among several units within a level. Since each decision making unit requires certain resources in order to carry on its activities, it is common for subordinate units to compete with each other for any resources which must be shared. In the structures to be studied in this research, the competition for scarce organizational resources will be a major source of the conflict between decision making units.

As several writers have noted [80, 105, 108] decentralization almost invariably involves some conflict of view between the central authority and lower decision making levels. As Simon states, "Decentralization can encourage the formation of and loyalty to subgoals which are only partly parallel to the goals of the organization" [106, p. 47]. Thus, the organization's objectives will seldom exactly coincide with the personal objectives of even those participants whose interest in the organization lies in the attainment of its goals [105, p. 114]. This statement is supported by empirical studies which are discussed in Machlup [77] and Williamson [123]. Furthermore, Smithies [108] has pointed out that although organizations may initially establish operating rules for subordinates, i. e., select objectives for subordinates to facilitate smooth functioning of the organization, in time a subordinate may adopt new objectives which reflect his own aspirations and goals. For example, Smithies states that, particularly in a bureaucratic organization, the desire for power often produces divergence of interest between subordinate levels and the organization as a whole.

An example of this conflict between subordinate decision making units has been reported by Simon [105, p. 201]. Simon's example, which was an actual incident that occurred in the California state government, demonstrates that two independent agencies subordinate to the

State Relief Administration (welfare department) could pursue their own objectives, but in fact contribute no value to the objectives of the State Relief Administration. In a business context Smithies [108, p. 11] illustrates conflict by discussing how the manager of the Chevrolet division of General Motors could be primarily interested in his own market share rather than the overall profit of GM.

Simon [103] has noted that one need not postulate conflict in personal goals between decision making units in order for conflict to arise. Conflicts can arise naturally because different parts of the organization are concerned with different activities, but these activities may affect other decision making units. To emphasize, Simon observes, "these conflicts could and would arise even if organizational decision making roles were being enacted by computers" [106].

Throughout this dissertation the term cooperative organization is applied when there is conflict between decision making units at a given level over limited resources, but there is no conflict between levels over objectives. Thus, in a two level cooperative organization with one superordinate and n subordinates, the objective function of the superordinate equals the sum of the subordinates' objective functions. The assumption is that each subordinate knows only his portion of the overall objective function. The superordinate may know the entire objective function, or he may assume the subordinates are using the correct objectives. The term non-cooperative organization applies to an organization where there also exists conflict between levels of the hierarchy over objectives.

To overcome these conflicts there appear to be two primary strategies:

(1) Change the organizational structure by decreasing the amount of decision making autonomy or by replacing and displacing decision making units.

(2) Develop a set of coordination mechanisms which can resolve conflicts and bring about decisions which are satisfactory to both the superordinate and to the subordinates. The concern of this dissertation is to study mathematically the system of controls which can be used to hold conflicts between decision making units within "tolerable" limits.

To summarize, the typical organization has a hierarchical structure. This research is concerned with a hierarchical organization whose resource allocation decisions are made at different levels and at different points within each level, thus yielding a decentralized structure. It is assumed that at specific levels in the organizational structure the resource allocation decision making process can be represented by a mathematical programming problem. The decentralization of decision making offers some appealing features, but it also creates control and coordination problems. This research mathematically explores the interpretation and implications of certain mechanisms for coordinating resource allocation decisions in both a cooperative and a non-cooperative organization. The next section investigates what types of coordination mechanisms exist, and how they function in a hierarchical decentralized organization.

Coordination Mechanisms

To facilitate the definition of coordination, consider a two level organization with one superordinate and n subordinates. Suppose further that each of the n+1 decision making units has certain goals and objectives which are mathematically reflected in the objective functions and constraint sets of each unit. The superordinate and n subordinates are arranged in a hierarchy, and thus the superordinate has some priority of action over each subordinate. This priority generally implies that the subordinates cannot make resource allocation decisions completely independent of the superordinate.

Given a set of resources available to the organization, the superordinate seeks a mechanism or a set of mechanisms through which he can influence the subordinates behavior so that they will undertake resource allocation programs which do not require more resources than are available, and which further his objective function. It is assumed that the organization's objectives are composed of the superordinate's objectives and the subordinates' objectives. Thus, unless the subordinates and the superordinate have the same aims, it is impossible to discuss the "organization's objectives" without recognizing the objectives of the superordinate and the subordinates. With respect to coordination Simon stated, "These mechanisms are aimed at adoption by all subordinates of nutually consistent decisions in combination attaining the superordinate's established goal," [105, p. 139]. Thus, for purposes of this thesis coordination mechanisms are devices by which the superordinate can influence the subordinates to seek a resource allocation program that furthers the objectives of the superordinate.

Mesarovic, Macko, and Takahara [84] define coordination in a similar manner. In their approach there are three sets of goals: the system's goals, the superordinate's goals, and the subordinates' goals. The superordinate's goal is to find means of influencing the subordinates' decision problems so that when each subordinate solves his decision problem, the system's objective function is optimized. Their analysis requires consistency among the different goals, i. e., the system's objective function must be optimized when the superordinate coordinates the subordinates, and each subordinate optimizes his problem [84, p. 97]. In this dissertation goal consistency is not required. Therefore, the concern is under what circumstances can the superordinate find coordination mechanisms which allow an optimal or satisfactory solution to be found.

Two specific characteristics of coordination mechanisms which are analyzed in later chapters are absolute and relative coordination. Absolute coordination refers to acts which influence the subordinates so that they find a resource allocation program which is optimal with respect to the superordinate's objective function, i. e., the subordinates arrive at the same program that the superordinate would have if he had complete information. Relative coordination is the term for acts which influence the subordinates to find a solution which is "satisfactory" but not necessarily optimal with respect to the objectives of the superordinate. It should be apparent that relative coordination is easier to attain than absolute coordination because of the analogy to satisficing versus optimizing.

The two primary classes of coordination mechanisms which are studied are:[2]

(1) goal intervention mechanisms, and

(2) constraint intervention mechanisms.

While these classes are distinctly different in the models to be studied, in actual organizational settings it is often difficult to identify pure forms of these intervention mechanisms.

Goal intervention methods include ways that a superordinate can influence the objective function of a subordinate. In its strongest form this might mean that the superordinate imposes a part of his overall objective function on the subordinate. In a less forceful form examples of goal intervention are incentives, tax allowances, bonuses, etc. [108]. Goal intervention methods also encompass the economist's pricing mechanism [80, p. 220].

Coordination through constraint intervention implies a method by which the feasible region of a subordinate is changed. Thus, methods which allow the superordinate to affect the amount of resources available to the subordinates (budgeting approaches) and methods which allow the superordinate to impose specific restrictions, e. g., you must not spend more than X dollars on a particular project, are examples of constraint intervention methods.

Although both coordination mechanisms have the same motives,

[2]These classes are similar to those mentioned by Sengupta and Ackoff [97, p. 12].

there are some significant differences in interpretation and application. For example, Simon [103] discusses how the goals of a decision making unit should be considered as the whole set of requirements (constraints) which are imposed on the unit. However, the final resource allocation decision is greatly affected by the subset of the requirements which are selected as the objective function.

Objectives and Overview of the Contents of the Dissertation

The purpose of this dissertation is to study, to present, and to analyze analytical models for resource allocation decision making in a hierarchical decentralized organization. The specific objectives that are accomplished include:

(1) The existing literature in economics, operations research, systems theory, and organizational behavior concerning models of the resource allocation decision making process in hierarchical decentralized organizations is integrated under a common framework. This integration indicates which analytical models are lacking in supporting empirical work.

(2) By drawing upon results and algorithms for optimization of large mathematical programming problems, the theoretical justification and validity for the two classes of coordination mechanisms when applied in a cooperative organization are studied. The results answer the following questions regarding the coordination mechanisms:

(a) Under what conditions can each of the two main coordination mechanisms be expected to work, i. e., bring about coordination with respect to the superordinate's goals?

(b) What are the economic and behavioral interpretations and implications of the different coordination mechanisms?

(c) What algorithms can be used to describe the coordination process?

(3) Extensions are suggested for certain algorithms to allow for more appealing economic and behavioral interpretations.

(4) An investigation is made of what happens when coordination mechanisms which work in a cooperative organization are used in a non-cooperative organization.

(5) A negotiation model is developed to demonstrate one feasible coordination process for a non-cooperative organization.

The dissertation is organized in seven chapters. Chapter I contains an introduction to the terms resource allocation decision making, hierarchical, decentralized, and coordination. The objectives of the research are also given.

Chapter II describes and defines the resource allocation decision process in a hierarchical decentralized organization. A mathematical model is presented which will be utilized to integrate and analyze existing methodology. Examples of applicability and significance of the results are given.

Chapters III and IV are devoted to a complete analysis of coordination mechanisms in a cooperative organization. Chapter III investigates goal intervention methods which are often called pricing methods. The existence conditions are stated, and a comparison and interpretation of the pricing mechanisms used in economics and systems theory is given.

A discussion of algorithms for goal intervention coordination is given, and some minor extensions are made for certain techniques.

Chapter IV treats coordination through constraint intervention in a cooperative organization. Geoffrion's treatise [46] on primal resource directive approaches is used as a foundation for the discussion. The major results of the chapter are the identification of existence conditions and an economic interpretation of Geoffrion's three basic approaches.

Chapter V begins by discussing the assumptions and implications inherent in coordination mechanisms for cooperative organizations. Specifically, it is shown that the structure of the organization has no effect on the final solution, the subordinates have no autonomy or influence on the final decision, and that relative coordination has no meaning in a cooperative organization. Next, it is shown that if coordination methods which worked in a cooperative organization are used in a non-cooperative organization that resource allocation programs can be found that do not use more than the supply of resources available. However, these programs will, in general, be non-optimal with respect to both the superordinate's and the subordinates' objective functions. In fact, programs which have very poor characteristics may be found.

Chapter VI proposes a negotiation model for conceptualizing the coordination of decisions in a non-cooperative organization. The coordination mechanism presented, called a goal partitioning procedure, finds resource allocation programs which do depend on the organization's structure. The procedure, which is interative, is shown to converge finitely to a solution (a set of resource allocation programs) that depends upon both the superordinate's and the subordinates' objectives. An interpretation for the behavioral implications of the procedure is given which demonstrates that the procedure has satisficing and compromising properties. Particular emphasis is placed on the kind of information which must be communicated during the goal partitioning procedure.

Chapter VII summarized the conclusions of the study by restating propositions which evolved in Chapters III to VI. Specific areas for future study are identified.

CHAPTER II

DESCRIPTION AND MATHEMATICAL REPRESENTATION OF THE RESOURCE ALLOCATION DECISION PROCESS

To assist in the ingegration and analysis of existing work in hierarchical decentralized decision making a description of a hypothetical resource allocation decision process is presented. Although the description is for a hypothetical process, it does agree with some existing verbal descriptions of such processes, e. g., in a socialist government's central planning agency [66, 118], in a profit making business organization [119], and in a federal government agency [14, 101]. Figure 1 depicts an abstraction of the kinds of information flows which often take place during the resource allocation decision process in a hierarchical decentralized organization.

Initially, there exists a set of inputs to the superordinate level. These inputs represent requirements or goals which are imposed on the organization by external forces, e. g., the stockholders may require that a certain profit level be achieved, or the federal government might require that certain standards be met. Other information known to the superordinate includes resources available such as money (possibly operating and capital budgets), manpower, facilities, etc. The superordinate also receives or possesses some information about the subordinates at the level ummediately below. Given these exogenous inputs, the information from the units below, and the superordinate's own expectations, the

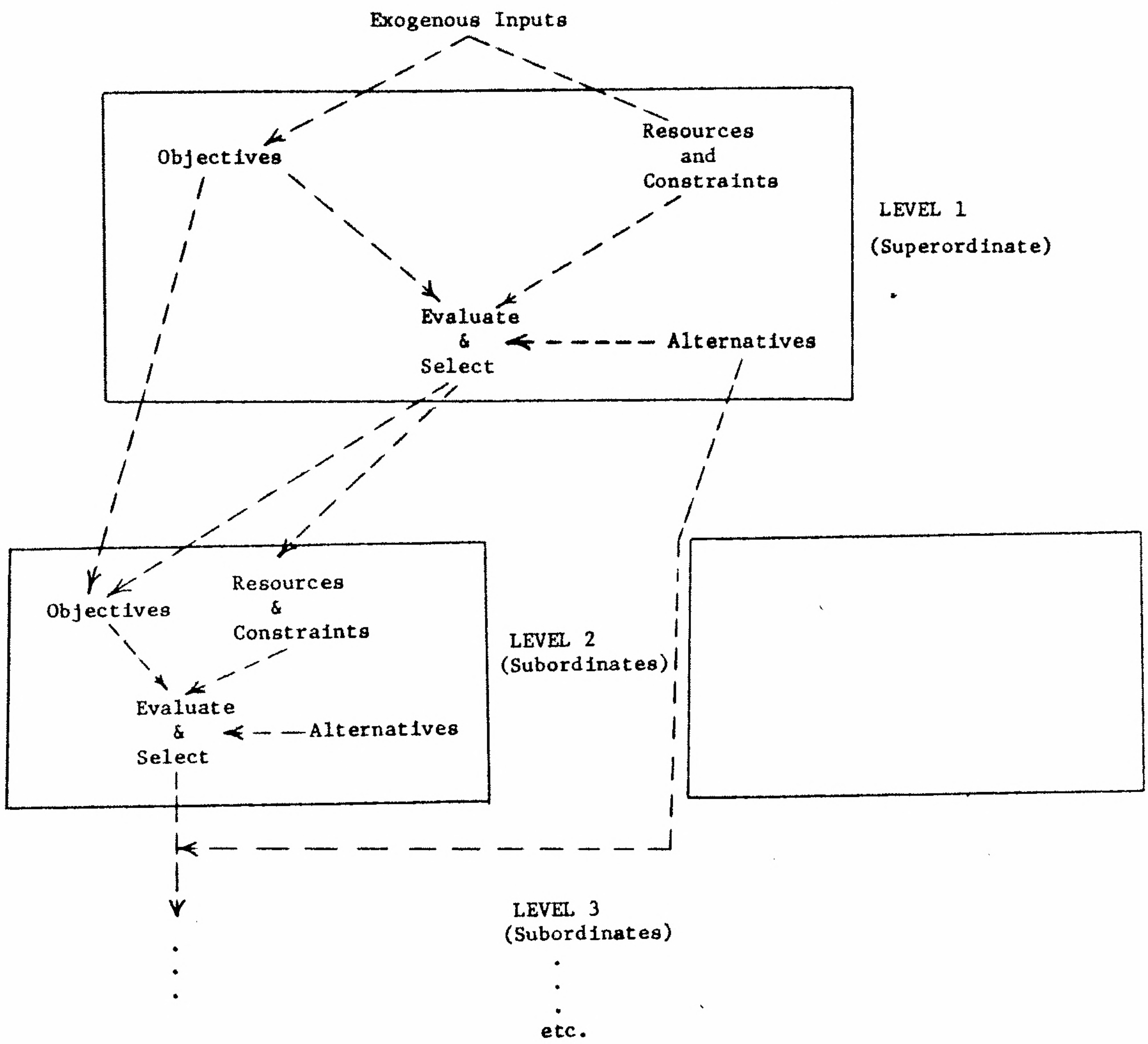

Figure 1. Information Flow in a Resource Allocation Process for a Hierarchical Decentralized Organization.

superordinate arrives at an objective (utility) function and a set of constraints. Based on the utility function and constraints, the superordinate tentatively selects a program of resource allocations. This program generally consists of a set of resource budgets for the subordinates in level two. These budgets and additional information for coordination purposes are communicated to the subordinate units. The additional information may be in the form of constraints, incentives, or other information about the superordinate's objective function and/or constraints.

A subordinate decision making unit at level two uses these inputs to arrive at an allocation plan in light of his own objectives and constraints. The entire information gathering, problem solution, and information communication process repeats itself for each of the subordinates at level two except the input from level one (the superordinate) replaces the exogenous inputs.

It is assumed, as shown in Figure 1, that there is no interaction between the subordinate decision units at a given level. This is typical of most existing models of decentralized decision making. However, often subordinates do interact with each other. Such interaction or interdependency is called a technological or behavioral externality and has been treated in the literature for special cases [56, 83, 94, 119].

The important aspects of this hypothetical process are:

(1) A decision unit's objectives and constraints are a function of the unit's expectations and capabilities, and information from levels above and below.

(2) The resources available to a decision unit are determined by

the next higher level. The superordinate's resources are affected by exogeneous factors.

(3) The evaluation and selection process for a unit is performed through consideration of its objective function and constraints.

(4) Subordinates at a given level compete for resources.

(5) In general a higher level unit does not concern itself with specific projects or activities which are undertaken by its subordinates, but rather concentrates on some aggregate measure of a subordinate's activity.

The resource allocation process is generally depicted as being iterative [14, 66, 101, 118]. The process just described is iterative in the sense that objectives and constraints may change stepwise as new information about alternatives becomes available. The iterative process begins with the superordinate selecting a program and then relaying the program along with coordination information to each subordinate at level two. Each subordinate solves its allocation problem and sends information about it along with coordination information to subordinates at level three. This process continues until the bottom levels are reached at which time the information now flows up through the hierarchy until it reaches the superordinate at level one. The upward flow of information is often called counter planning [66]. At each decision unit the upward flow from subordinates is integrated and aggregated so that at the very top the superordinate has summary information about what would happen if the tentative budgets assigned at the beginning of the cycle were actually realized. The superordinate uses this information to arrive at a new set of tentative budgets, and the entire process repeats itself. After a finite number of cycles, the final allocations are determined, and the final plan is implemented. Descriptions of similar such processes can be found in [14, 66, 93, 100, 118, 119, 122].

Framework for Analysis: A Mathematical Model

To facilitate the accomplishment of the research objectives given in Chapter I and to allow a more rigorous analysis and discussion of the decision process, a mathematical model is now presented. The model is for a hypothetical two level organization. A two level organization is used because it is easier to describe and conceptualize, and it seems to capture the important characteristics of the resource allocation decision process. Suppose the organization is arranged as in Figure 2, i. e., there is one superordinate (or supremal) and n subordinate (or infimal) decision making units. The superordinate's decision problem is select vectors, $\underline{\alpha}_i$ $(i = 1, \ldots, n)$ in order to

$$\text{maximize } \Phi(g_1(\underline{x}_1), \ldots, g_n(\underline{x}_n)) \tag{2-1}$$

$$\text{subject to: } \sum_{i=1}^{n} \underline{\alpha}_1 \leq \underline{b} \tag{2-2}$$

$$\sum_{i=1}^{n} \underline{\Psi}_i(\underline{\alpha}_i) \leq \underline{n} \tag{2-3}$$

where the decision problem facing subordinate i is to select decision

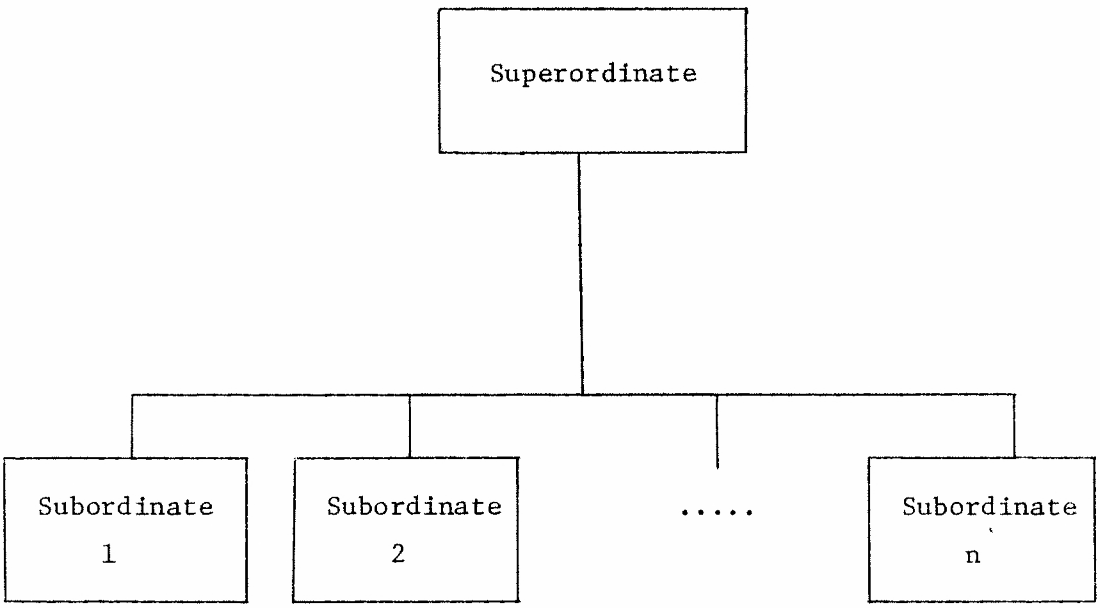

Figure 2. A Two Level Organization Chart.

variables $\underline{x}_i$ in order to

$$\text{maximize} \quad f_i\,(\underline{x}_i) \tag{2-4}$$

$$\text{subject to:} \quad \underline{h}_i(\underline{x}_i) \leq \underline{\alpha}_i \tag{2-5}$$

$$\underline{x}_i \in X_i \tag{2-6}$$

Clearly, these problems are not well defined in the usual sense of mathematical optimization. The intent at present is to describe or represent mathematically a very general process. A major difficulty with the above model is the form is murky, i. e., it is unclear (and in fact not defined) what the objective for the organization is.

Throughout this dissertation all vectors are columnar and are designated by an underline, e. g., $\underline{x}$. Transposes are represented by a prime, e. g., $\underline{x}'$. The superordinate has a vector, $\underline{b}$, of m resources, which can be used by the n subordinates. Expression (2-2) indicates that the sum of the resources used by all subordinates must not exceed the supply. Further, there may exist additional constraints, e. g., like expression (2-3), concerning how the subordinates can use the resources, e. g., it might be required that the budget for subordinate i be no greater than the budget for subordinate j. Given a set of organization resources which must be shared by the subordinates and possibly constraints on how these resources are to be used (these constraints are imposed by forces outside the organization), the superordinate must decide, or influence the subordinates to decide, how to allocate these resources to the subordinates in order to maximize or satisfice his objective function, expression (2-1). The argument for the superordinate's objective is some measure of effectiveness, $g_i(\underline{x}_i)$, for each subordinate. Since the subordinates actually carry out the activities which bring benefit to the superordinate, the superordinate's objective function value depends on "how well" the subordinate does. However, in the general case subordinate i tries to attain his goals by maximizing his objective, $f_i(\underline{x}_i)$, whose value is a function of the vector of activity levels, $\underline{x}_i$. Thus, it is possible for the subordinate to pursue his own objective, (2-4), and not further the superordinate's objective. This points up the possibility of conflict in the organization.

While subordinate i pursues his objective, he must choose a vector of activity levels, $\underline{x}_i$, which satisfies certain restrictions, (2-5) and (2-6). Constraint (2-5) is a vector function which specifies that the subordinate can use no more organizational resources than he has available. Constraint (2-6) forces subordinate i to select only those $\underline{x}_i$ which are within subordinate i's feasible region. These constraints are common only to subordinate i and represent technological restrictions.

This model is quite general and allows one to represent both goal intervention and constraint intervention coordination mechanisms. The important points to be noted about the model are:

(1) Both the superordinate and the subordinates have their own set of goals, i. e., objectives functions and constraints. It is possible for the superordinate and the subordinates to pursue different

objectives because the superordinate's objective function may not be related to the subordinate's objective functions.

(2) Explicitly the model indicates that the superordinate impacts on the subordinates through the selection of the $\underline{\alpha}_i$'s because $\underline{\alpha}_i$ represents the resources available to subordinate i. It will be shown that in order to coordinate the activities of the subordinates, the superordinate may have to employ other techniques such as influencing the form of the objective function or imposing constraints. These additional techniques tend to decrease the subordinate's decision making autonomy and increase the amount of information which must be communicated during the allocation process.

(3) The subordinate has an impact on the behavior of the superordinate through $g_i(\underline{x}_i)$ because $\underline{x}_i$ is determined by the subordinate.

(4) The superordinate need not have detailed information about the constraints or objective functions of the subordinates.

There are a number of different strategies that one could pursue in analyzing coordination mechanisms with respect to the general mathematical model. The ploy used in this dissertation is to first consider a very special case of the general model. However, almost all previous analytical analysis has dealt with this special case. Chapters III and IV treat in detail the case when the superordinate's objective function equals the sum of the subordinates' objective functions, i. e.,

$$\Phi(g_1(\underline{x}_i),\ldots, g_n(\underline{x}_n)) = f_1(\underline{x}_1) + \ldots + f_n(\underline{x}_n).$$

An organization in which this relationship holds is referred to as a "cooperative" corganization because there is no conflict between the superordinate's and the subordinates' objectives. In this environment the concept of coordination has to do with choosing parameters (prices, resources, etc.) such that the overall function is maximized.

Before beginning the mathematical analysis, it is pertinent to indicate what relevance this research has, and to what kinds of organizations it is applicable. Therefore, the remainder of this chapter addresses these questions.

Relevance of this Study and Some Examples

Many organizations have hierarchical structures and a decentralized allocation process; consequently, there exists the need to coordinate resource allocation decisions. For the most part there has been little work on the construction and analysis of conceptual analytical models of the coordination process. As Zannetos [126, p. 51] has noted, the problems of designing and restructuring an organization are for the most part unsolved. Those in organization behavior tend to ignore the economic aspects, while those in economics tend to ignore the human element. Managers and organizational designers need new tools to study formally the effects of structure on the decision process. Ansoff and Brandenberg state, "If we knew how to construct analytical models of organizations, the objectives of the organization could be used also as a criteria of organizational design." [7, p. 709] The analytical models which are developed and analyzed in this dissertation are a beginning.

Specifically, the author feels that the results of the research are important contributions because

(1) While the analytical models are theoretical, they do lend validity to certain kinds of coordination mechanisms in specific environments. As Malinvaud states, "As always should be the case, a formal theoretical study must bring a better understanding of practical questions"[78, p. 171].

(2) The framework developed provides a basis whereby various coordination mechanisms can be compared, contrasted, and evaluated for specific environments.

(3) The study provides mathematical precision in defining both concepts and issues.

(4) The results suggest a starting point for some empirical studies.

It is felt that the concepts used in this research have wide application: in industry, government, etc. As an indication of the generality of applications of coordination mechanisms in hierarchical decentralized organizations consider the following simple examples:

Example 2.1: Profit Making Organization

Consider a business which manufactures goods for customers. The objective for the organization is to maximize overall corporate profit. The organization is divided into three divisions, and each division operates independently reporting to the corporate headquarters. Thus, the company is organized into three separate profit centers. A typical statement about the structure might be similar to the following which Whinston [120, p. 407] cites, "The direct responsibility for managing company line operations rests with the general manager of each division. Under our form of organization, each division represents a separate profit center for the purpose of management control, and the general manager is accountable for earning a satisfactory rate of return on the assets employed in his operation." The corporate headquarters is primarily interested in long term profit with other objectives being, maintaining a certain share of the market, providing a quality product, achieving a conservative growth, etc. Each division manager also has certain objectives which include division profit, division market share, etc. There is not necessarily complete agreement among all divisions and the corporate headquarters about the objectives or the relative importance of the different objectives. The problem then is, how can the corporate headquarters coordinate the activities of the divisions so that the corporate's objectives are met, and the division still maintains some decision making authority.

Example 2.2: Educational Institution

Consider a state university system which is composed of a four year non-technical institution, a technical institute, and several two year junior colleges. Each of these entities has certain goals and objectives, e. g., the junior colleges might be primarily interested in providing an "adequate" liberal arts education to every state resident who desires one. The non-technical institution might be concerned with providing an "adequate" non-technical education to those state residents who can meet certain entrance standards, performing research which would benefit the state, and maintaining a graduate program. The technical institute has such goals as becoming a nationally recognized institution for graduate education and providing a "good" technical education for highly qualified students. Clearly, the objectives and priorities of these three classes differ. In addition, the state legislature has certain objectives which it believes the state schools should accomplish. These objectives may not always agree with those of the institutions. The state legislature allocates resources to the schools for their activities. The problem is how can the legislature influence the behavior of the institutions to further the legislature's goals.

Example 2.3: Federal Government - City Government Revenue Sharing

Suppose the federal government has decided to decentralize some of its support programs. The decision has been made to share with the cities a portion of the revenue which is generated by taxes. The federal government hopes this will cut out some of the bureaucratic red tape, etc. and will allow those officials closer to the problems to utilize the money more effectively. However, there are some general goals which the federal government would like to have the cities pursue with the shared revenue. The problem is, how can the federal government share revenue with the cities and still be assured that an attempt is made to accomplish the federal government's objectives.

These three examples from three different environments were given to illustrate the general relevance of the research results. The research deals with a model which is intentionally very general because results are sought which do not depend upon the unique characteristics of any one type of organization. While the analytical models and theoretical framework developed in this research are probably not applicable at an operational level, it is believed that this work can indicate general directions and provide useful insights.

CHAPTER III

COORDINATION THROUGH GOAL INTERVENTION IN A COOPERATIVE ORGANIZATION: PRICING APPROACHES

The purpose of this chapter is to present and analyze methods of goal intervention for coordination when the superordinate's objective function equals the sum of the subordinates' objective functions (a cooperative organization). Under this assumption, goal intervention methods are often called pricing approaches.[3] Other authors [96, 113] have used the term "indirect distribution" to describe the same process. Throughout this dissertation a pricing approach implies a goal intervention method applied to a cooperative organization. This special case arises whenever the superordinate has the power to delegate an objective function to which the subordinate must adhere.[4]

It is important to emphasize that the author's interest lies primarily in the economic and behavioral interpretations of pricing approaches. Particular emphasis is placed on the applicability of pricing <u>as a conceptualization of</u> coordination in a decentralized organization.

[3] In the mathematical theory of decomposition, procedures which are identical to the pricing approaches to be presented are often called price directive techniques [46, 54, 119]. The term price directive is avoided here because of possible misinterpretations.

[4] The concern here is not with how or when such an event is possible, a subject which has been discussed by Hertz [57] and Smithies [108]. Clearly, in some cases such as example 3 of Chapter II (the revenue sharing plan) the superordinate does not have the power to set the subordinates' objective functions.

This chapter begins by a straightforward derivation of the pricing mechanism from the method of Lagrange multipliers. Since in nearly all discussions of decentralization through prices in the economic literature the classical work of Koopmans and Arrow and Hurwicz is cited, a brief discussion of their approaches is given. Next, the economic interpretation for pricing is given, and the relationships between the economist's pricing scheme and the balanced pricing scheme from systems theory is discussed. Finally, the existence conditions for prices are stated and proven, and algorithms for finding the correct prices are discussed.

Decentralization Through Prices

In general the existing work treats the problem of subordinate i as:

$$\underset{\underline{x}_i \in X_i}{\text{Maximize}} \qquad f_i(\underline{x}_i)-\underline{\lambda}_i'\underline{h}_i(\underline{x}_i)$$

where $\underline{\lambda}_i$ is a vector of prices or penalties. These prices serve as a means of influencing the relative weight associated with the subordinate's objective function without actually fixing an entirely new objective function. The superordinate's task is to choose $\underline{\lambda}_i$ for $i = 1,\ldots,n$ so that the allocation plans selected by the subordinates are optimal as well as feasible for the organization. Thus, the name "price" coordination arises from the interpretation of $\underline{\lambda}_i$ as a set of prices associated with organizational resources.

To see how prices are used as a means of coordination for the general formulation given in Chapter II under the assumption

$$\Phi(g_1(\underline{x}_1), \ldots, g_n(\underline{x}_n)) = f_1(\underline{x}_1) + \ldots + f_n(\underline{x}_n)$$

notice that the superordinate's and the subordinates' problems can be expressed as:

Problem P

$$\text{maximize} \sum_{i=1}^{n} f_i(\underline{x}_i) \tag{3-1}$$

subject to:

$$\sum_{i=1}^{n} \underline{h}_i(\underline{x}_i) \leq \underline{b} \tag{3-2}$$

$$\sum_{i=1}^{n} \underline{\Psi}_i(\underline{h}_i(\underline{x}_i)) \leq \underline{n} \tag{3-3}$$

$\underline{x}_i \in X_i$ for $i=1,\ldots,n$

and $\underline{x}_i$ $(i=1,\ldots,n)$ is an element of the positive orthant of Euclidean ρ_i space.

Note that it has been implicitly assumed that $\underline{\alpha}_i = \underline{h}_i(\underline{x}_i)$. With respect to interpretation and theoretical validity, nothing is lost by such an assumption. Thus, with price coordination it is no longer necessary to consider $\underline{\alpha}_i$ as a decision variable. It should be clear that under the assumption of a cooperative organization the problem given in (2-1) to (2-6) is well defined, i. e., (2-1) is the objective function and (2-2), (2-3), (2-5), and (2.6) are constraints. Also, it is clear that if

$\underline{x}_i$* (i=1,...,n) is optimal for problem P, then $\underline{x}_i$* and $\underline{\alpha}_i$* = $\underline{h}_i(\underline{x}_i$*) is optimal for the problem given in Chapter II.

In order to solve problem P by having subordinate i solve the part of problem P which depends only on his decision variables, the coupling constraints[5] (3-2) and (3-3) must be suppressed. However, the final solution must satisfy these constraints, and therefore it is the superordinate's task to ensure that this is accomplished. The ploy used for doing this is to move (3-2) and (3-3) into the objective function. This scheme is simply the Lagrangian approach in classical optimization. Assuming for the present that there exists a coordinating set of prices, $\underline{\lambda}_1$, $\underline{\lambda}_2$, the result is another problem which can be written as:

$$\text{maximize} \quad \sum_{i=1}^{n} f_i(\underline{x}_i) - \underline{\lambda}_1' \left[\sum_{i=1}^{n} \underline{h}_i(\underline{x}_i) - \underline{b}\right] - \underline{\lambda}_2'\left[\sum_{i=1}^{n} \underline{\Psi}_i(\underline{h}_i(\underline{x}_i)) - \underline{\eta}\right]$$

$$\underline{x}_i \in X_i \qquad i=1,\ldots,n$$

or equivalently

$$\text{maximize} \quad \sum_{i=1}^{n} \left(f_i(\underline{x}_i) - \underline{\lambda}_1\underline{h}_i(\underline{x}_i) - \underline{\lambda}_2' \, \underline{\Psi}_i(\underline{h}_i(\underline{x}_i))\right) + \underline{\lambda}_1'\underline{b} + \underline{\lambda}_2' \, \underline{\eta} \qquad (3\text{-}4)$$

$$\underline{x}_i \in X_i \qquad i=1,\ldots,n$$

where $\underline{\lambda}_1$ and $\underline{\lambda}_2$ are pricing vectors. For a given $\underline{\lambda}_1$ and $\underline{\lambda}_2$ the problem given in (3-4) can be solved by having subordinate i solve the following problem:

[5]So called because, they link the subordinates together.

Problem $S_i(\lambda)$

$$\text{maximize} \quad f_i(\underline{x}_i) - \underline{\lambda}_1'\underline{h}_i(\underline{x}_i) - \underline{\lambda}_2'\underline{\Psi}_i(\underline{h}_i(\underline{x}_i))^{6}$$

$$\underline{x}_i \in X_i$$

One can associate an iterative scheme for arriving at a set of prices, which when communicated to a subordinate allows him to find his portion of the optimal solution to problem P (assuming for the moment that such prices exist) with an information exchange scheme between superordinate and subordinates. Such an iterative process would begin by having the superordinate choose a $\underline{\lambda}_1$ and $\underline{\lambda}_2$ and announce them to the subordinates. Subordinate i solves problem $S_i(\lambda)$ and communicates some information, usually $f_i(\underline{x}_i)$, $\underline{h}_i(\underline{x}_i)$, and $\underline{\Psi}_i(\underline{h}_i(\underline{x}_i))$ to the superordinate. The superordinate uses this information in some way to arrive at a new set of prices which he announces to the subordinates. After some number (possibly infinite) of these information exchanges, the subordinates find the optimal solution to problem P.

Notice the savings in the amount of information which must be communicated under this scheme versus a centralized process. In a centralized process all information about $\underline{h}_i(\underline{x}_i)$ and $\underline{\Psi}_i(\underline{h}_i(\underline{x}_i))$ for all $\underline{x}_i \in X_i$ $(i=1,\ldots,n)$ must be communicated to the superordinate who then solves problem P by himself. In the decentralized scheme using

---

[6]Notice that $\underline{\lambda}_1'\underline{b} + \underline{\lambda}_2'\underline{n}$ can be dropped because it is a constant for fixed $\underline{\lambda}_1$ and $\underline{\lambda}_2$.

prices only summary information need be transmitted. For example, it is normally not necessary for subordinate i to transmit $\underline{x}_i$ to the superordinate at each iteration.

A brief summary is now given for the famous works of Koopmans and Arrow and Hurwicz. Their work was among the first to analytically consider decentralization through prices.

<u>Koopmans [64,65]</u>

This work is often cited [38, 90] as the first definitive work on mathematical analysis of decentralized decision making. Koopmans work was concerned with activity analysis of an entire economic system. His work is related to general equilibrium theory and welfare economics and is rooted in some famous works in classical economics [17, 71, 74, 87]. The essential strategy is to develop the concept of a "proper" set of prices through which functions of delegated management may be performed.

Given an input output matrix, $A = [A_F \vdots A_I \vdots A_P]$ which relates the activity level vector, $\underline{x}$, to the commodity vector $\underline{y}' = [\underline{y}_F', \underline{y}_I', \underline{y}_P']$ where $\underline{y}_F$ = final commodity components, $\underline{y}_I$ = intermediate commodity components and $\underline{y}_P$ = primary commodity components, one seeks $\underline{x}$ so that $\underline{y}_F \geq 0$, $\underline{y}_I = 0$, and $-\underline{y}_P \leq -\underline{S}_P$. Assuming such a $\underline{y}$ exists, a point $\hat{\underline{y}}$ is said to be "efficient" in the sense of Koopmans if and only if there does not exist any other point $\underline{y}$ with the property

$$\underline{y}_F \geq \underline{y}_F \qquad \text{and} \qquad \hat{\underline{y}}_F \neq y_F \, .$$

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[7] It is assumed that primary commodities are supplies, thus $\underline{y}_P \leq 0$, and therefore $\underline{S}_P$ represents the amounts of primary commodities available.

If an efficient $\hat{\underline{y}}$ exists then from linear programming theory there exists a dual price vector $\underline{p}' = (\underline{p}_F', \underline{p}_I', \underline{p}_P')$. Koopmans then shows that a necessary and sufficient condition for the efficiency of a vector $\underline{y}$ is that there exist a vector $\underline{p}$ such that

$$\underline{p}'\underline{y} = 0 \tag{3-5}$$

$$\underline{p}'A \leq 0 \tag{3-6}$$

$$\underline{p}_F \leq 0, \ \underline{p}_P' \ (\underline{y}_P - \underline{S}_P) = 0 \tag{3-7}$$

One can interpret each column of the A matrix, $\underline{a}_K$ as being controlled by a manager. Thus, when $\underline{p}'\underline{a}_K > 0$ the manager in charge of activity K should expand because in terms of opportunity costs this expansion will lead to a better $\underline{y}_F$. On the other hand, whenever $\underline{p}'\underline{a}_K < 0$ the cost of expanding this activity exceeds the benefits it can produce.

Charnes and Cooper [29, p. 296] have written Koopman's model in a slightly different way which illuminates Koopmans' reasoning. The model is:

maximize $\underline{v}_F' \, \underline{y}_F$

subject to: $A_P\underline{x} = \underline{y}_P \geq \underline{S}_P$

$A_I\underline{x} = \underline{y}_I = \underline{0}$

$$A_F \underline{x} = \underline{y}_F \geq \underline{0}, \quad \underline{x} \geq 0 .$$

The dual of this model is

$$\text{minimize} \qquad -\underline{S}_P' \, \underline{p}_P$$

$$\text{subject to:} \qquad \begin{aligned} \underline{p}_P' A_P + \underline{p}_I' A_I + \underline{p}_F' A_F &\leq \underline{0} \\ \underline{p}_P' &\leq 0 . \end{aligned}$$

Here the dual variables appear as prices for the commodities $\underline{y}$. Conditions (3-5) to (3-7) are derived by observing the complementary slackness conditions in the above pair of linear programs.

The decentralized process imagined by Koopmans for finding an efficient state involves a "helmsman" who sets the value for each final commodity; a custodian for each intermediate or primary commodity who sets tentative prices for them and changes the prices to attain equilibrium between supply and demand, and then buys or sells all that is asked for at this price; and a manager for each activity who tries to maximize his profit with the given prices.

Koopmans argues that this process has only one solution which is the optimal solution. He also implies that these dynamic rules could lead to convergence. However, the dynamic aspect of the rules is vague, and no convergence proof is given [10].

This work was very significant in that it recognized how a decentralized scheme could be used to do planning for an entire economic system in which each firm in the economy tries to maximize its own profit. As Koopmans says, "the decentralization utilizes incentives that are naturally operative in the market system." [64, p. 22] His work also points up the remarkable feature of a purely competitive economic system which is: a completely centralized control mechanism cannot do better in terms of an overall utility function than a decentralized competitive system.

However, as March and Simon [80, p. 202] point out, this result has little relevance for decentralization through pricing within the firm because in a firm the conditions for perfect competition are not met. In the absence of external markets to provide prices, the organization suffers all the problems associated with monopoly and imperfect competition [80, p. 202].

Arrow and Hurwicz [10]

Suppose an organization faces the following resource allocation problem:

$$\text{Maximize} \qquad U(y_1,\ldots,y_n)$$

$$\text{subject to:} \qquad \sum_{j=1}^{m} g_{ij}(x_j) + \varepsilon_i - y_i \geq 0 \text{ for } i=1,\ldots,n$$

$$\sum_{j=1}^{m} g_{ij}(x_j) + \varepsilon_i \geq 0 \text{ for } i=n+1,\ldots,S$$

where U = the utility function of the organization,

y_i = amount of final product i to be produced,

ε_i = amount of product i available initially,

x_j = scale of division j's production, and

g_{ij} = amount of product i produced by the jth division.

If one writes the Lagrangian function associated with this problem and then rearranges the terms, the results are:

$$L(y,x:p) = U(y_1,\ldots,y_n) + \sum_{i=1}^{n} p_i \left(\sum_{j=1}^{m} g_{ij}(x_j) + \varepsilon_i - y_i\right)$$

$$+ \sum_{i=n+1}^{S} p_i \left(\sum_{j=1}^{m} g_{ij}(x_j) + \varepsilon_i\right)$$

$$= [U(y_1,\ldots,y_n) - \sum_{i=1}^{n} p_i y_i] + \sum_{j=1}^{n} [\sum_{i=1}^{S} p_i g_{ij}(x_j)] + \sum_{i=1}^{S} p_i \varepsilon_i$$

Given a value for p_i (i=1,...,S), the maximization of L(y,x;p) which is a sum of functions each depending on a different set of variables, involves maximizing each of the functions separately. This suggests the decentralization concept.

Each division is instructed to maximize $\sum_{i=1}^{S} p_i g_{ij}(x_j)$ for a given p_i where p_i is interpreted as the price of commodity i. At the same time the central authority determines the level of final demands by maximizing the difference between utility, $U(y_1,\ldots,y_n)$ and costs $\sum_{i=1}^{n} p_i y_i$. As Arrow and Hurwicz state, "The elements of decentralization are clear. For a given set of prices, a process (division) manager need know only the prices and the technology of his own process. The helmsman (central authority) need only know the prices of desired commodities (products) and the utility function." [10, p. 77].

Of course, it must be shown that there in fact does exist some set of prices, $p_i (i=1,\ldots,n)$ which when the Lagrangian is maximized yields the optimal y_i and x_j. Arrow and Hurwicz suggest but do not prove existence conditions. These conditions are that $U(y_1,\ldots,y_n)$ be a strictly concave function, and that the constraints are concave and satisfy a constraint qualification.

Arrow and Hurwicz also devise a price adjustment mechanism based on a gradient method and show that under the conditions given above, their procedure converges to an optimal solution. Thus, they were among the first to propose an algorithm for conceptualizing the resource planning process from a decentralized point of view.

Economic Interpretations

There is an interesting economic interpretation which can be given to the coordination through pricing scheme. Suppose problem P represents an organization's resource allocation decision problem where the objective is to maximize profit. Thus, $f_i(\underline{x}_i)$ is the contribution to organizational profit by division (subordinate) i by undertaking activities $\underline{x}_i$. The restriction $\underline{x}_i \in X_i$ represents the requirements and restrictions which are common only to division i. The constraints, $\sum_{i=1}^{n} \underline{h}_i(\underline{x}_i) \leq \underline{b}$, are restrictions on the amounts of organizational resources available. These resources are to be shared between the divisions. The constraints, $\sum_{i=1}^{n} \underline{\Psi}_i(\underline{h}_i(\underline{x}_i)) \leq \underline{\eta}$, are restrictions on the way the organizational sources can be allocated, e. g., exogeneous forces might specify that division one should never receive a higher budget than division two.

The components of $\underline{\lambda}_i$ and $\underline{\lambda}_2$ used in coordinating the divisions are frequently called Lagrange multipliers, dual variables, shadow prices, or imputed costs. These components have the effect of giving an imputed cost to the coupling constraints. For example, Zangwill [125, p. 66] shows that the k^{th} component of $\underline{\lambda}_1$ yields a measure of how valuable a small increase of the k^{th} resource would be to the organization. Furthermore, the components of $\underline{\lambda}_1$ can be thought of as prices. If the supply of the k^{th} resource is not exceeded, i. e., $\sum_{i=1}^{n} \underline{h}_i^k (\underline{x}_i) < \underline{b}^k$, then its price $\lambda_1^k = 0$. On the other hand if $\sum_{i=1}^{n} h_i^k(\underline{x}_i) = b^k$, then λ_i^k may be non-zero which can be interpreted as a charge or bonus made by the organization against any division which uses the k^{th} resource. Therefore, the organization accomplishes coordination by having a superordinate charge subordinates (and thus affect the attractiveness of certain activities) for use of a resource if the demand for that resource exceeds the supply.

The components of $\underline{\lambda}_2$ are also prices. They are concerned with the constraining effect of the constraints dealing with the manner in which resources are used. For example, suppose an organization consists of two subordinate decision units. Let x_{ij} be the dollar support allocated to project j under the control of subordinate i. Because of extraneous factors there is a restriction imposed on the organization which specifies that subordinate 1 should never have more dollar support for his projects than subordinate 2. In addition, suppose there is only a limited supply of dollar support for the use of the subordinates. Thus, in problem P for this organization the following two constraints appear:

$$\sum_{j=1}^{p_1} x_{1j} + \sum_{j=1}^{p_2} x_{2j} \leq b \qquad (3\text{-}8)$$

$$\sum_{j=1}^{p_1} x_{1j} - \sum_{j=1}^{p_2} x_{2j} \leq 0 \qquad (3\text{-}9)$$

Let λ_1 be the price associated with (3-8) and λ_2 be the price associated with (3-9). Suppose at some iteration of the information exchange process, the superordinate specifies that $\lambda_1 = .1$ and $\lambda_2 = .8$. Each superordinate is charged .10 for every dollar he uses in resources (this can be interpreted as a cost of capital). In addition, subordinate 1 is charged \$.8 more for every dollar used because of the additional constraint (3-9). Likewise subordinate 2 is rewarded \$.8 per dollar spent. Although a subordinate may not explicitly know about the constraints dealing with how resources must be used, the existence of additional prices like $\lambda_2 = .8$ in the above example do convey information about these constraints. For instance, in the above example subordinate 1 would have an indication that there are restrictions at a higher level relating to the amount of resources he can use. Therefore, the components of $\underline{\lambda}_1$ and $\underline{\lambda}_2$ can be viewed as incentives (both positive and negative) for increasing or decreasing the amounts of resources allocated to an activity.

In terms of information, the central unit supplies only a vector of prices which is the same for each division. Hence, there is a great savings in the amount of information which must be transmitted between divisions of the organization over what would be required in a centralized organization. However, in some cases (in particular when the overall objective is linear) it can be shown that this price information alone is insufficient for coordination to be achieved. In the next section a slightly different method is proposed which supplies additional information for coordination purposes.

The primary application of price coordination mechanisms has been in the determination of transfer prices for goods which are sold between divisions in the same company. For a detailed discussion, see [30, 39, 53, 58, 86, 100].

Balanced Pricing

This pricing mechanism has been suggested in the literature of systems theory [84, 96, 99] as a way to solve a large optimization problem. It is often referred to in the literature as the interaction balance principle, and it has been particularly useful in applying decentralized control procedures to large industrial processes such as chemical and steel processes.

The logic of the balanced pricing mechanism is similar to the pricing scheme discussed earlier; however, there are some differences, particularly in terms of economic interpretation. In problem P it was noted that (3-2) and (3-3) are the links between the subordinates. To simplify notation it is assumed that the only coupling constraints are included in (3-2). Suppose the linking connections between the subordinates are cut by introducing additional variables and constraints,

i. e., (3-2) can be written as

$$\underline{h}_i(\underline{x}_i) + \underline{z}_i \leq \underline{b} \qquad (3\text{-}10)$$

for i=1,...,n

$$\underline{z}_i = \sum_{\substack{j=1 \\ j\neq i}}^{n} \underline{h}_j(\underline{x}_j) \qquad (3\text{-}11)$$

$\underline{z}_i$ is now a vector of decision variables under the control of subordinate i, and the links between the subordinates are expressed in (3-11). As before a vector of Lagrange multipliers, $\underline{\mu}_i$, is assigned to the constraints in (3-11), and these constraints are moved from the constraint set into the objective function. Thus, the overall objective function is:

$$\text{maximize} \quad \sum_{i=1}^{n} f_i(\underline{x}_i) - \sum_{i=1}^{n} \underline{\mu}_i' \left(\sum_{\substack{j=1 \\ j\neq i}}^{n} \underline{h}_j(\underline{x}_j) - \underline{z}_i \right)$$

$$\underline{x}_i \in X_i \quad (i=1,\ldots,n)$$

One can separate this objective function into n parts where the i^{th} part depends only on $\underline{x}_i$ and $\underline{z}_i$. The resulting decision problem for subordinate i is:

Problem $SB_i(\mu)$

$$\text{maximize} \quad f_i(\underline{x}_i) + \underline{\mu}_i'\underline{z}_i - \sum_{\substack{j=1 \\ j\neq i}}^{n} \underline{\mu}_j'\underline{h}_i(\underline{x}_i) \qquad (3\text{-}12)$$

subject to: $$h_i(\underline{x}_i) + \underline{z}_i \leq \underline{b} \qquad (3\text{-}13)$$

$$\underline{x}_i \in X_i$$

The following remark which is really a statement of the interaction balance principle given as a definition of coordinatability by Mesarovic, Macko and Takahara [84, p. 100] shows the relationship between the optimal solutions to the subordinate's problem (problem $SB_i(\mu)$) and the optimal solutions for problem P.[8]

Remark 3.1

If an optimal solution for problems $SB_i(\mu)$ $(i=1,\ldots,n)$ has the property that $\underline{z}_i = \sum_{\substack{j=1 \\ j\neq i}}^{n} \underline{h}_j(\underline{x}_j)$ $(i=1,\ldots,n)$ then that solution is optimal for problem P.

The proof is trivial. Remark 3.1 implies that if a vector, $\underline{\mu}_i$, $i=1,\ldots,n$ could be found so that the optimal solutions to SB_i, $i=1,\ldots,n$, satisfy (3-11) then the organization's decision problem is solved. The superordinate's task is to choose $\underline{\mu}_i$ for $i=1,\ldots,n$. Thus, the $\underline{\mu}_i$'s serve as coordination mechanisms which the superordinate is free to manipulate.

For a general two level organization with n subordinates, the balance principle results in an increase in the number of decision variables of an amount equal to the number of coupling constraints for each subordinate's problem.

The procedure for finding the optimal prices, $\underline{\mu}_i$, (assuming

[8]The assumption is that constraints like (3-3) are not present.

of course that they exist) could be iterative, and a procedure similar to the information exchange given in the previous section could be used. Thus, the superordinate passes down prices, and receives some information about the solution to each of the subordinates' problems. Although the iterative procedure for finding the "balanced" prices is much the same as before, the economic interpretation for them is different. Lasdon and Schoeffler [73] allude to the interpretation of prices using the interaction balance principle, however to the author's knowledge no one has pointed out their difference from the economist's prices and the significance of the difference.

Suppose that an organization's objective function is to maximize profit. As Mesarovic, Macko and Takahara [84] point out, under balanced pricing the subordinate's problem (problem $SB_i(\mu)$) has constraint (3-13) whereas the subordinate's problem (problem $S_i(\lambda)$) before did not. In effect, the superordinate is allowing the subordinate to share in making the organization's resource allocation decision. For example, subordinate i has decision variables $\underline{x}_i$ and $\underline{z}_i$. $\underline{x}_i$ is a vector of activity levels while $\underline{z}_i$ represents the vector of organizational resources which are to be made available to the other subordinates. The objective function for subordinate i, (3-12) is composed of three components. $f_i(\underline{x}_i)$ is again the contribution of subordinate i's activities to organizational profit. The component $\sum_{\substack{j=1 \\ j\neq i}}^{n} \mu_j{}'\underline{h}_i(\underline{x}_i)$ is the charge made against subordinate i for consuming organizational resources. This component is exactly like the term $\underline{\lambda}_1{}'\underline{h}_i(\underline{x}_i)$ with the pricing scheme discussed earlier. The third component, $\underline{\mu}_i{}'\underline{z}_i$ represents the contribution by the rest of the organization to organizational profit. Thus, the K^{th} component of $\underline{\mu}_1$ represents the contribution to profit per unit of the K^{th} resource used by the other n-1 subordinates. Therefore, subordinate i's objective is to maximize the contribution to profit from all subordinates minus a penalty for using organizational resources to pursue his own activities.

In terms of information, the superordinate communicates to each subordinate two vectors of prices. One vector, $\sum_{\substack{j=1 \\ j \neq i}}^{n} \underline{\mu}_j$ represents the prices to be paid by subordinate i for the use of organizational resources. The other vector, $\underline{\mu}_i$, represents a measure of the aggregate profit per unit of resource allocated to the rest of the organization. In balanced pricing the pricing vectors communicated to each subordinate may be different, whereas with the other pricing mechanism, the information communicated to each subordinate was the same. Also, the subordinate has available more information for his decision, i. e., he now knows something about how his efficiency compares to that of the rest of the organization. This additional information can provide a method of "alternative testing" [103, p. 7] because it allows the subordinate to compare his contribution to profit against the contribution to profit of the rest of the organization. Thus, such information could serve as an impetus for the subordinate's improvement.

Coordination through balanced pricing seems much like load-type coordination as described by Messarovic, Macko and Takahara [84, p. 59]. They describe load-type coordination as the case when subordinate units explicitly recognize the existence of other decision units on the same level and the superordinate unit provides the subordinate units with a model of the relationship between its action and the response of the system.

The information communicated upward from the subordinate to the superordinate consists only of an aggregate allocation for itself and for the rest of the organization. For example, subordinate i could send to the superordinate values of $f_i(\underline{x}_i)$, $\underline{h}_i(\underline{x}_i)$ and $\underline{z}_i$. Thus, the subordinate is not specifying how the resources to be allocated to the other subordinates, $\underline{z}_i$, is to be divided between them.

In comparison to the pricing scheme described in the previous section, balanced pricing provides and, in fact, requires that more information be communicated both upward and downward. Also, balanced pricing results in a larger (in terms of the number of variables and constraints) subordinate's problem than the other pricing scheme.

Other than the possible behavioral advantages associated with a balanced pricing scheme, the question remains as to whether coordination through balanced pricing works under more general conditions. Unfortunately, it does not appear that the additional communication required in balanced pricing does not make the existence conditions more general as the next section will show.

Existence Conditions for Pricing

This section answers the question of, under what mathematical conditions can one guarantee that a set of prices exists which allow the superordinate to coordinate the activities of the subordinates. In other words, when can one be assured that there exists $\underline{\lambda}_1$* and $\underline{\lambda}_2$* such that when these prices are announced to the subordinates, the subordinates will solve problem $S_i(\lambda^*)$ and find an optimal solution to problem P with no additional information from the superordinate. The strategy taken is to briefly sketch the historical developments with respect to the existence conditions and to provide some motivation, through examples, for the discussion. The theoretical development is restricted to the pricing scheme given initially on pages 36 - 38. However, analogous results can be given for the case of balanced pricing.

The reader should recognize that the concern is with existence conditions for pricing for decentralization. The discussion is not intended to focus on issues of price directive approaches for solving mathematical programming problems, e. g., see [20, 54].

Koopmans [65], Arrow and Hurwicz [10], Uzawa [114], and Whinston [119] have all suggested that in problem P if the objective function (3-1) is strictly concave, and the constraints are convex and satisfy a constraint qualification, then coordinating prices do exist. However, the author has been unable to find a formal proof given to support this hypothesis in their work. Recently, Moeseke and Ghellinck [85] and tenKate [113] have given formal proofs for similar existence statements. The proofs rely heavily on the Kuhn-Tucker saddlepoint and stationary point conditions.[9]

The first rigorous analysis of the price existence question seems to have been given by Charnes, Clower, and Kortanek [30]. They proposed the following theorem.

Theorem 3.2 (Charnes, Clower, and Kortanek [30])

Suppose an optimal solution $\underline{x}_i{}^*$ $(i=1,\ldots,n)$ exists for problem P, and suppose the following conditions are met

[9] See Mangasarian [79] for a detailed discussion.

(a) $f_i(\underline{x}_i)$ is strictly concave on X_i

(b) constraints (3-2) and (3-3) are linear,

then there exists $\underline{\lambda}_1$* and $\underline{\lambda}_2$* such that when the subordinate solves problem $S_i(\lambda^*)$, an optimal solution is $\underline{x}_i$*. The proof given by Charnes, et al. [30, p. 304] uses arguments from semi-infinite programming. The assumption in theorem 3.2 of linearity avoids certain problems which can occur and which require satisfaction of a constraint qualification.[10]

Theorem 3.3 allows one to ascertain whether some solution, $\underline{x}_i$ (i=1,...,n) is optimal with respect to the organization's objective function, (3-4).

<u>Theorem 3.3 (Lasdon [72, p. 84])</u>

If given some $\underline{\lambda}_1^* \geq \underline{0}$, $\underline{\lambda}_2^* \geq \underline{0}$, $\underline{x}_1^*$: i=1,...,n is optimal for problem S_i: i=,...,n and if

$$\sum_{i=1}^{n} \underline{h}_i(\underline{x}_i^*) \leq \underline{b},$$

$$\sum_{i=1}^{n} \underline{\Psi}_i(\underline{h}_i(\underline{x}_i^*)) \leq \underline{n},$$

$$\underline{\lambda}_1^{*\prime} \left(\sum_{i=1}^{n} \underline{h}_1(\underline{x}_i^*) - \underline{b} \right) = 0, \text{ and} \tag{3-14}$$

$$\underline{\lambda}_2^{*\prime} \left(\sum_{i=1}^{n} \underline{\Psi}_i(\underline{h}_i(\underline{x}_i^*)) - \underline{n} \right) = 0, \tag{3-15}$$

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[10] See Mangasarian [79, p. 94].

then $\underline{x}_i^*$, i=1,...,n is optimal for problem P.

Proof: By hypothesis $\underline{x}_i^*$, i=1,...,n is feasible for problem P. Since $\underline{x}_i^*$ is optimal for problem S_i, then

$$f_i(\underline{x}_i^*) - \underline{\lambda}_1^{*\prime}(\underline{h}_i(\underline{x}_i^*)) - \underline{\lambda}_2^{*\prime}(\underline{\Psi}_i(\underline{x}_i^*)) \geq f_i(\underline{x}_i) - \underline{\lambda}_i^{*\prime}(\underline{h}_i(\underline{x}_i)) - \underline{\lambda}_2^{\prime *}(\underline{\Psi}_i(\underline{h}_i(\underline{x}_i))$$

$$\text{for all } \underline{x}_i \in X_i$$

Thus, summing over all i and using (3-14) and (3-15) gives

$$\sum_{i=1}^{n} f_i(\underline{x}_i^*) \geq \sum_{i=1}^{n} f_i(\underline{x}_i) - \underline{\lambda}_1^{\prime *}\left[\sum_{i=1}^{n} \underline{h}_i(\underline{x}_i) - \underline{b}\right] - \underline{\lambda}_2^{*\prime}\left[\sum_{i=1}^{n} \underline{\Psi}_i(\underline{h}_i(\underline{x}_i)) - \underline{\eta}\right]$$

$$\text{for all } \underline{x}_i \in X_i$$

but

$$\sum_{i=1}^{n} h_i(\underline{x}_i) \leq \underline{b} \qquad \text{and} \qquad \sum_{i=1}^{n} \underline{\Psi}_i(\underline{h}_i(\underline{x}_i)) \leq \underline{\eta}$$

must be satisfied, therefore

$$\sum_{i=1}^{n} f_i(\underline{x}_i^*) \geq \sum_{i=1}^{n} f_i(\underline{x}_i)$$

for all $\underline{x}_i$ feasible to problem P. Thus, $\underline{x}_1^*$, i=1,...,n must be optimal for problem P.

The converse of the theorem is false as demonstrated by tenKate

[113, p. 741]. The reason related to the possibility of non-convexities in the PR space which is the cartesian product of the objective function value and the constraint function space. These problems have been analyzed in detail by Falk [44].

Theorem 3.3 provides a mechanism for ensuring that an optimal solution has been found. The result is appealing because no convexity requirements are needed. However, since in general the converse is false, if the superordinate was to use the conditions in theorem 3.3 as a basis for stopping the information exchange procedure, the iterative process might never end. This is true because without additional restrictions the conditions of theorem 3.3 are sufficient but not necessary for an optimal solution to problem P.

The converse of theorem 3.3 can be proven if the hypothesis is strengthened.

Theorem 3.4 (Mangasarian [79])

If x_i^* (i=1,...,n) is an optimal solution to problem P and if the following conditions hold

(a) $f_i(\underline{x}_i)$ is concave for $x_i \varepsilon X_i$ (i=1,...,n),

(b) $\underline{h}_i(\underline{x}_i)$ and $\underline{\Psi}_i(\underline{h}_i(\underline{x}_i))$ are convex vector functions on X_i, (i=1,...,n)

(c) and (3-2) and (3-3) satisfy a constraint qualification,

then conditions (3-14) and (3-15) hold.

The proof is a direct result of the Kuhn-Tucker saddlepoint necessary optimality theorem [79, p. 79]. The results of theorems 3.3 and 3.4 can provide for the superordinate a mechanism for determining whether a solution, $\underline{x}_i$ (i=1,...,n), found by the subordinates, is optimal for the organization's resource allocation problem. The only information that the superordinate needs is $\underline{h}_i(\underline{x}_i*)$ and $\underline{\Psi}_i(\underline{h}_i(\underline{x}_i*))$ for i=1,...,n, and an assurance that subordinate i maximized his portion of the objective function. Thus, it is not necessary that the superordinate know the objective function of each subordinate.

So far, no assertions have been made about when the prices $\underline{\lambda}_1*$ and $\underline{\lambda}_2*$ actually exist. One might suspect that the existence of coordinating prices is tied directly to the Kuhn Tucker saddle point necessary conditions, i. e., concavity of objective, convexity of constraints, and satisfaction of a constraint qualification. However, as Baumol and Fabian [19], and Charnes, Clower, and Kortanek [30] have pointed out if problem P is linear, it may be impossible to coordinate using prices. Remember that being able to coordinate means that if a subordinate is given pricing vectors, $\underline{\lambda}_1$ and $\underline{\lambda}_2$, he can find his portion of the optimal solution without any additional information. The failure in linear problems can be attributed to the existence of multiple solutions to the subordinate's problem. In mathematical decomposition theory this is not a problem because additional procedures such as taking convex combinations can be used. However, from a decentralization interpretation this is a problem. Theorem 3.5 rules out these alternative optima.

Theorem 3.5 (Moeseke and Ghellinck [85, p. 75]

Suppose an optimal solution $\underline{x}_i*$ (i=1,...,n) for problem P exists. If the following conditions hold

(a) $f_i(\underline{x}_i)$ is strictly concave on X_i,

(b) (3.2) and (3.3) satisfy a constraint qualification,[11] and

(c) $\underline{h}_i(\underline{x}_i)$ and $\underline{\Psi}_i(\underline{h}_i(\underline{x}_i))$ are convex on the convex set X_i.

Then there exists vectors of prices, $\underline{\lambda}_1$* and $\underline{\lambda}_2$*, such that if subordinate i solves problem $S_i(\lambda^*)$ he finds $\underline{x}_i$*.

The existence of multiple alternative optima to the subordinate's problem, $S_i(\lambda^*)$ for the optimal coordinating price can be a problem when only some of the optima are optimal for problem P. Theorem 3.5 avoids this difficulty by requiring f_i to be strictly concave, and thus have a unique maximum. As it is shown shortly a linear form for problem P often results in multiple optima.

Obviously, the above theorems prove only the existence of prices and do not indicate how to find the optimal price vectors, $\underline{\lambda}_1$*, and $\underline{\lambda}_2$*.[12] However, the conditions in the theorem's hypothesis do say something about when decentralized resource allocation decision making in a hierarchy can be coordinated by the pricing mechanism. First, condition (a) in theorem 3.5 requires that the objective function of each subordinate have decreasing returns to scale. Thus, constant returns to scale (linear) do not qualify. Condition (c) requires that as the decision variables for a subordinate change, the change in a <u>constraint function</u> must take place at least as rapidly as a constant

[11]Note that problems $S_i(\lambda)$ are not subject to a constraint qualification [85, p. 75].

[12]It should be clear that these optimal prices are the dual variables associated with the dual of problem P.

times the decision variable.

In an actual organization one could get some idea of whether coordination through pricing might have hope of succeeding by examining the form of its objectives and technological restrictions. Although it is likely that most organizations do not explicitly quantify the objective function and constraints and treat their resource allocation problem as an optimization problem, the results of this section indicate that only under quite restrictive conditions can coordination through prices be achieved. Perhaps this explains why there appear to be few examples of where a "real world" organization coordinates through the pricing mechanism.

The hypothesis of theorem 3.5 clearly rules out an interesting class of problems, viz., linear problems, because linear functions are not strictly concave. However, since the optimal coordinating prices are simply the optimal dual variables, and since optimal dual variables for linear problems are well understood, one might wonder what the difficulty is. The following example shows that in a linear problem if the optimal dual variables are used as prices, the subordinates' problems can have multiple alternative optimal solutions.

Example 3.2

Suppose the organization's resource allocation problem is given by

$$\text{maximize} \quad 2X_1 + X_2$$

$$\text{subject to:} \quad \begin{aligned} X_1 + X_2 &\leq 1 \\ X_1 \quad &\leq 2 \\ X_2 &\leq 3 \\ X_1, X_2 &\leq 0 \end{aligned}$$

The optimal solution to this problem is $X_1^* = 1$, $X_2^* = 0$. The usual ploy is used to eliminate the coupling constraint. The problem now becomes

$$\text{maximize} \quad 2X_1 + X_2 - \lambda(X_1 + X_2 - 1)$$

$$\text{subject to:} \quad \begin{aligned} X_1 \quad &\leq 2 \\ X_2 &\leq 3 \\ X_1, X_2 &\geq 0 \end{aligned}$$

The subordinates problems are now

$$\text{maximize} \quad 2X_1 - \lambda X_1 \quad \text{and maximize} \quad X_2 - \lambda X_2$$

$$\text{subject to:} \quad X_1 \leq 2 \qquad \text{subject to:} \quad X_2 \leq 3$$

$$X_1 \geq 0 \qquad\qquad\qquad X_2 \geq 0$$

From duality theory it can be shown that $\lambda^* = 2$. However, from the decentralization viewpoint $\lambda^* = 2$ causes subordinate 1's objective function to vanish. Therefore, given $\lambda^* = 2$ subordinate 1 is indifferent toward any feasible solution, i.e., $0 \leq X_1 \leq 2$. In terms of mathematical decomposition subordinate 1 can be forced to select $X_1 = 2$. However, in order to do so some additional information other than a price must be communicated. This additional information violates the thesis's assumption regarding decentralization through pricing.

In example 3.2 the reason that both subordinates cannot find the overall optimal solution by prices alone is because of the linearity in the problem. In general, it is impossible to achieve coordination with respect to the superordinate's objective function for a linear problem.

Recall that the balanced pricing scheme appeared to provide for the communication of more information than the other pricing scheme. This might lead one to suspect that balanced prices might exist under more general conditions. The author has found no statement concerning existence conditions for balanced prices, but it can be easily shown that the results in theorem 3.5 also hold for balanced pricing. In addition the author has been unable to show that balanced pricing does work under more general conditions.[13]

Algorithms for Finding Coordinating Prices

As it was mentioned in the previous section the coordinating prices are really the dual variables associated with the coupling constraints in problem P. The theory of mathematical programming also is concerned with finding these prices because there is a strong analogy between solving the organization's problems by

[13] In particular, linear models have been extensively investigated and the conclusions are the same as with the other pricing scheme.

breaking it into n parts, assigning a subordinate to each one, and decomposing a large mathematical programming problem. In fact, the development of decomposition approaches has supplied most of the algorithms which can be interpreted as procedures for describing the coordination process in hierarchical decentralized organizations.

It is not the purpose of this reaearch to discuss at length all or even most of the decomposition algorithms which have been proposed. There exist excellent discussions from a mathematical standpoint in [54, 72]. However, to demonstrate that procedures exist by which a superordinate can coordinate the activities of subordinates in certain cases, some representative algorithms are discussed. With each of the algorithms to be mentioned, close attention is paid to the economic interpretation for the procedure.

Dantzig-Wolfe [37, 38]

Dantzig and Wolfe were the first to propose an algorithm which could be interpreted as representing decision making in a decentralized structure for a linear problem. Although a linear problem does not satisfy the existence conditions a pricing approach can be used to find the optimal solution. However, the subordinate decision makers cannot make the final decision. To show this and to illustrate the algorithm, consider the problem facing an organization comprised of two divisions. Division 1 can produce any of p_1 products while division 2 can produce any of p_2 commodities. In order to produce a product a division requires organizational resources, e. g., money, manpower, etc. In addition, there may exist constraints on the way organizational resources are used. The constraints on the supply of organizational resources and the way they are used can be expressed via

$$H_1\underline{x}_1 + H_2\underline{x}_2 \leq \underline{b} .$$

In addition, each division has technological and other restrictions on its production capacity which are expressed as: $A_i\underline{x}_i \leq \underline{C}_i$.

The linear programming problem for this situation is:

$$\text{maximize} \qquad P = \underline{f}_1{}'\underline{x}_1 + \underline{f}_2{}'\underline{x}_2$$

$$\begin{aligned} \text{subject to:} \quad & H_1\underline{x}_1 + H_2\underline{x}_2 \leq \underline{b} \\ & A_1\underline{x}_1 \qquad\qquad \leq \underline{C}_1 \\ & \qquad\quad A_2\underline{x}_2 \quad \leq \underline{C}_2 \\ & \qquad\quad \underline{x}_1, \underline{x}_2 \quad \geq 0 \end{aligned}$$

where $\underline{f}_i = p_i \times 1$ vector of profit coefficients

$H_i = m \times p_i$ matrix

$\underline{x}_i = p_i \times 1$ vector of outputs for division i

Let $S_i = \{\underline{x}_i \mid A_i\underline{x}_i \leq \underline{C}_i, \underline{x}_i \geq 0\}$ and suppose S_i is bounded.[14] Any point in S_i can be written as a convex combination of extreme points [72], i. e., if $\underline{x}_i \in S_i$ then $\underline{x}_i = \sum_j \lambda_i^j \underline{x}_i^j$, $\lambda_i^j \geq 0$, $\sum_j \lambda_i^j = 1$, and $\underline{x}_i^j$ is an extreme point of S_i. Thus, the above problem can be written as:

$$\text{maximize} \qquad \sum_j (\underline{f}_1{}'\underline{x}_1^j)\lambda_1^j + \sum_K (\underline{f}_2{}'\underline{x}_2^K)\, \lambda_2^K$$

---

[14] This assumption can be relaxed [38, p. 773], but it is made here to simplify developments.

$$\text{subject to:} \quad \sum_j (H_1\underline{x}_1^{\,j})\lambda_1^j + \sum_K (H_2\underline{x}_2^{\,K})\lambda_2^K \leq \underline{b} \tag{3-16}$$

$$\sum_j \lambda_1^j = 1 \tag{3-17}$$

$$\sum_K \lambda_2^K = 1 \tag{3-18}$$

$$\lambda_1^j, \quad \lambda_2^K \geq 0$$

This problem is the coordinator's problem and has decision variables, λ_1^j and λ_2^K. Using a column generating procedure, the $\underline{x}_1^j$ and $\underline{x}_2^K$ vectors are generated as needed. The usual simplex optimality criterion requires that

$$\min_j \; (H_1\underline{x}_1^j)' \, \underline{\pi}_o + \pi_1 - \underline{f}_1'\underline{x}_1^j \;\geq 0$$

and

$$\min_K \; (H_2\underline{x}_2^K)' \, \underline{\pi}_o + \pi_2 - f_2'\underline{x}_2^K \;\geq 0$$

where $\underline{\pi}_o$, π_1 and π_2 are the dual variables associated with (3-16), (3-17), and (3-18), respectively. Thus, in order to choose λ_1^j to enter the basis, the subproblem

$$\text{maximize} \quad \underline{f}_1'\underline{x}_1 - (H_1\underline{x}_1)'\underline{\pi}_o = (\underline{f}_1 - H_1\underline{\pi}_o)'\underline{x}_1$$

$$\text{subject to:} \quad A_1\underline{x}_1 \leq \underline{C}_1$$

$$\underline{x}_1 \geq \underline{0}$$

are solved by the division one. The economic interpretation is that the divisions are charged by the central unit for usage of the organizational resources, $\underline{\pi}_o$ being a vector of prices.

Figure 3 illustrates the iterative information exchange which takes place between the coordinator (the superordinate) and the divisions (subordinates). Initially, the coordinator assigns a price to each coupling constraint. Each subordinate uses these prices to solve its divisional problem, and then transmits to the coordinator two pieces of information: the current objective function value excluding the penalty term $\underline{\pi}_o' H_K$, and $H_K \underline{x}_K$. It is not necessary to transmit $\underline{x}_K$. The coordinator uses this information to arrive at a new pricing vector, $\underline{\pi}_o$, and the process begins anew. It can be guaranteed that at each stage the overall solution's objective function value increases and that the process converges in a finite number of iterations. The algorithm proceeds like the two phase simplex procedure. In phase I an overall feasible solution is sought via penalties for infeasibilities [38, p. 772-773]. As soon as a feasible solution is found in the master problem, phase II begins. In phase II the algorithm is primal feasible, so that if the iterative process stops, the organization has a feasible but not necessarily optimal solution. As it can be shown this feature distinguishes the Dantzig-Wolfe approach from most other pricing mechanisms which are dual feasible methods.

As it was demonstrated in the previous section, often there does not exist a set of prices, $\underline{\pi}_o$, which allow each division to find its part of the optimal solution on its own. Hence, the Dantzig-Wolfe

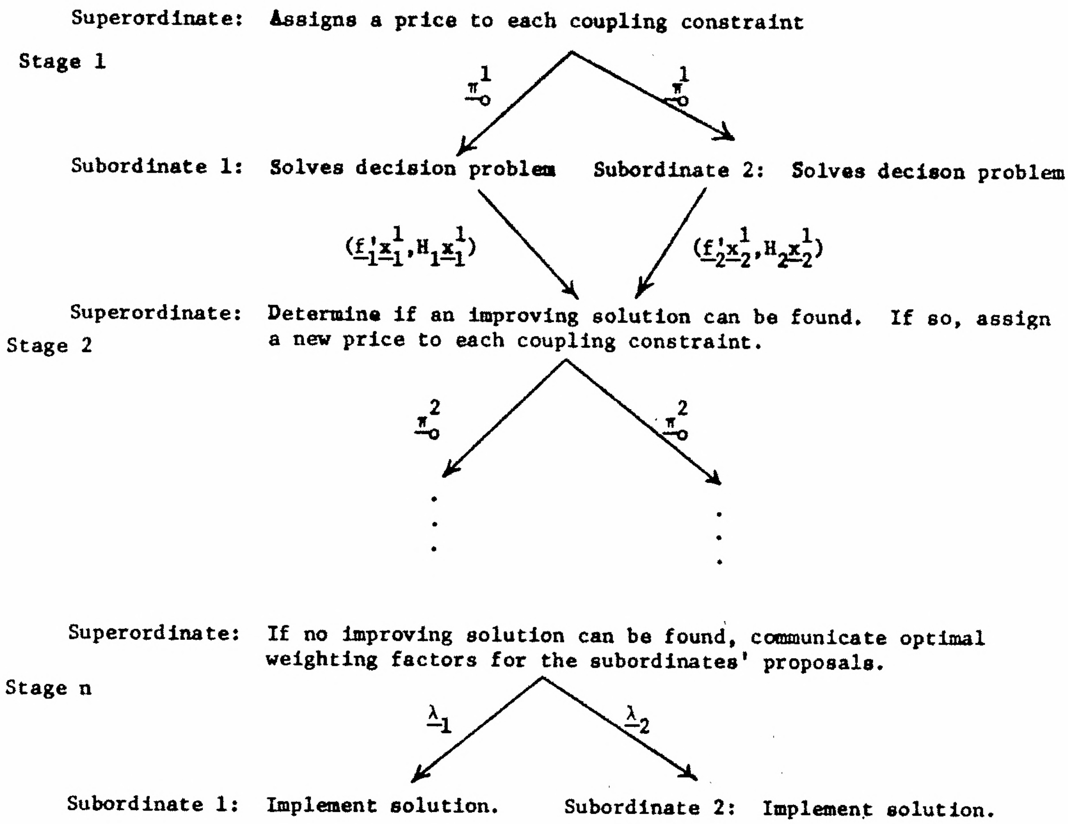

Figure 3. Information Exchange Process Associated the Dantzig-Wolfe Procedure.

algorithm cannot be conceptualized as decentralized because in the end the superordinate unit makes the final decisions by assigning optimal weights, $\underline{\lambda}_1^j$ and $\underline{\lambda}_2^K$, to the divisional proposals. Dantzig [36] refers to this scheme as centralized planning without complete information.

This situation is behavorially distressing because the subordinate is not allowed to make the final decision himself. Although he provides information to the decision process, i. e., he sends proposals to the superordinate, he is not allowed to choose the final allocation plan. Thus, the Dantzig-Wolfe process does not take advantage of the motivational potential of a decentralized structure. The superordinate treats the subordinates as if they were information processors. This philosophy would seem to correspond with Taylor's idea of scientific management [80].

To alleviate this drawback Charnes, Clower and Kortanek [30] have proposed what they call "pre-emptive goals". This scheme allows the superordinate to supply the subordinates with additional information, viz, a pre-emptive goal. In their work a division's problem is given by:

$$\text{maximize} \quad \underline{f}_i'\underline{x}_i - (H_i\underline{x}_i)'\underline{\pi}_o - M \,||\, \underline{\gamma}_i^* - H_i\underline{x}_i \,|| \tag{3-19}$$

$$\text{subject to:} \quad A_i\underline{x}_i \leq \underline{C}_i$$

$$\underline{x}_i \geq \underline{0}$$

where M = a large positive number, and

$\underline{\gamma}_i^*$ = a vector of organizational resources which would be used by division i under the optimal overall solution.

The pre-emptive goal formulation forces each division to get as close as possible to a solution which has $||\underline{\gamma}^* - H_i\underline{x}_i|| = 0$. Hence, the pre-emptive goal is to have the j^{th} component of $H_i\underline{x}_i$ equal to the j^{th} component of $\underline{\gamma}_i^*$. Of all solutions which satisfy this requirement, the subordinate attempts to choose that one which maximizes $\underline{f}_i{}'\underline{x}_i - (H_i\underline{x}_i)'\underline{\pi}_o$. It is not clear why the term $(H_i\underline{x}_i)'\underline{\pi}_o$ is required because the method can work without it. Unfortunately, such a scheme implies that the superordinate knows the solution to the overall problem, i. e., he must know how much resource each subordinate should get. [33, p. 52].

Although Collomb [33] feels that this requirement is too restrictive to be useful, the optimal pre-emptive goal $\underline{\gamma}_i^*$ can be found easily via the Dantzig-Wolfe algorithm. Instead of delegating the optimal weights to divisional proposals as the final step in the Dantzig-Wolfe algorithm, one can assign the pre-emptive goal $\underline{\gamma}_i^* = H_i\underline{x}_i^*$, where $\underline{x}_i^*$ is the overall optimum found by using the optimum weights. In effect the superordinate knows what decision the subordinate will make, but he allows the subordinate to calculate it on his own. One should also recognize that (3-19) is not linear. This problem can be converted to a linear one by using a simple goal formulation, i. e., make $\underline{\gamma}_i^* = H_i\underline{x}_i$ a constraint and assign large penalties for positive and negative deviations for not meeting this constraint. Viewed in this way, pre-emptive goals are no longer pricing coordination mechanisms, but they are a form of coordination through constraints, which is discussed in the next chapter.

Balas [15]

Balas has also proposed an algorithm for finding coordinating prices when problem P is linear. The algorithm has been discussed elsewhere [20, 54]. His approach is similar to the Dantzig-Wolfe algorithm, but he chooses the pricing vector differently. Specifically, Balas begins with the optimal solution to each division's problem ignoring the coupling constraints. If this solution is feasible for the coupling constraints, then it is optimal overall.

If it is not, then a "linking program" is solved which minimizes the amount of infeasibility while still maintaining optimality in the sense of the simplex optimality criterion, i. e., dual feasibility is maintained. Thus, unlike phase I of the Dantzig-Wolfe algorithm which moves toward feasibility as rapidly as possible, Balas constrains the move to ensure overall optimality. Thus, the approach is somewhat analogous to the primal dual algorithm in linear programming.

Balas presents two versions of an algorithm for accomplishing his purpose. Although in his article he purports that his algorithm can be used to bring about decentralized planning, both versions of his approach require the superordinate to have much more information than is required by other coordination procedures. It is necessary for the superordinate to know the current optimal basis for each division when he chooses a new pricing vector. Thus, while Balas's algorithm may offer computational advantages in solving large linear programming problems, it is not an appealing procedure for conceptualizing the coordination process in a decentralized hierarchical organization because it assumes that the superordinate has extensive information about the activities of the subordinates.

Other Algorithms [14, 19, 55, 56, 72, 119]

A number of modifications and extensions have been made in the Dantzig-Wolfe approach. Whinston [119] and Baumol and Fabian [19] relaxed the assumption that all relationships must be linear. They show that as long as the divisional constraints yield bounded convex regions, then non-linearities are allowed in the divisional constraints. Whinston also shows the limitation of a price system when the overall problem is not separable. In this conjunction Whinston generalizes division K's constraints to the form

$$\underline{a}_K(\underline{x}_K;\ \underline{x}_t) \leq \underline{c}_K \tag{3-20}$$

where $\underline{x}_t$ is under the control of division t. Thus, the division's activities are interdependent. This is called an externality and can be handled by introducing an additional variable, $\underline{s}_t$ so that (3-20) is now

$$\underline{a}_K(\underline{x}_K;\ s_t) \leq \underline{c}_K$$

An additional constraint $\underline{x}_t = \underline{s}_t$ is added to the coordinator's problem and treated as a coupling constraint. However, as Ruefli points out [90, p. 56] this can result in multiple prices being generated.

Using Wolfe's simplex method for quadratic programming [124] and

Dorn's duality theory for quadratic programming [40], Hass [55,56] has developed a decomposition approach for problem P when the objective function is quadratic and the constraints are convex. He also allows a simple kind of externality in the objective function. Hass's algorithm can be used as a step by step process description of coordination through pricing when the objective function is quadratic.

It is well understood that the pricing mechanism derived earlier using the Lagrangian is a dual approach because the prices are the dual variables. Lasdon [72, pp. 396, 458] and Grinold [54] present an excellent discussion of the theory and algorithms for these methods. Their discussion is not repeated here; however, it is useful for later work (in Chapter V) to present a very rudimentary algorithm due to Uzawa [114] and discussed by Jennergren [60].

Suppose the organization's problem is given by problem P. Uzawa's price adjustment rule searches for an optimal price vector through the following two step procedure:

(1) The superordinate announces a tentative set of pricing vectors, $\underline{\lambda}_1^t$ and $\underline{\lambda}_2^t$. For a given $\underline{\lambda}_1^t$ and $\underline{\lambda}_2^t$ subordinate i solves its problem (problem $S_i(\lambda^t)$). Subordinate i then communicates to the superordinate the vectors, $\underline{h}_i(\hat{\underline{x}}_i^t)$ and $\underline{\Psi}_i(\underline{h}_i(\hat{\underline{x}}_i^t))$, where $\hat{\underline{x}}_i^t$ is optimal for problem $S_i(\lambda_i^t)$.

(2) The superordinate computes a new price vector $\underline{\lambda}_1^{t+1}$ and λ_2^{t+1} as follows:

$$\lambda_{1j}^{t+1} = \text{maximum } [0, \lambda_{1j}^{t} + B(\sum_{i=1}^{n} h_{ij}(\hat{\underline{x}}_i^t) - b_j)]$$

$$\lambda_{2j}^{t+1} = \text{maximum}\ [0,\ \lambda_{2j}^{t} + B\ (\sum_{i=1}^{n} \Psi_{ij}(\underline{h}_i(\hat{\underline{x}}_i^{t}) - \eta_j)]$$

where λ_{Kj}^{t} = the j^{th} component of $\underline{\lambda}_K$ at iteration t,

$h_{ij}(\hat{\underline{x}}_i^{t})$ = the j^{th} component of $\underline{h}_i(\underline{x}_i^{t})$ at iteration t,

$\Psi_{ij}(\underline{h}_i(\underline{x}_i^{t}))$ = the j^{th} component of $\Psi_i(\underline{h}_i(\underline{x}_i^{t}))$ at iteration t, and

B = a "sufficiently small" [113, p. 156] positive scaler.

Iteration t+1 is initiated by the superordinate announcing $\underline{\lambda}_1^{t+1}$ and $\underline{\lambda}_2^{t+1}$. Given that the hypothesis of theorem 3.5 is satisfied, it can be shown that this algorithm converges to the optimal set of coordinating prices and hence to the optimal solution to problem P.

It should be noted that the information flows required for execution of the above price adjustment rule are nominal. The superordinate announces a set of prices for use by all the subordinates. Each subordinate then communicates information about $\underline{h}_i(\underline{x}_i)$ and $\underline{\Psi}_i(\underline{h}_i(\underline{x}_i))$. It is not necessary to send information about the current objective function value. Thus, the superordinate need not concern himself with the objective function.

Recently, Jennergren [60, 61] has presented an algorithm which can be used to conceptualize the coordination procedure for a linear problem. The method uses price schedules instead of pricing vectors. Since it is known that in a decentralized setting, a pricing coordination mechanism by itself cannot work for a linear program, Jennergren used duality theory to show how a set of coordinating price schedules could work. In effect these pricing schedules cause a subordinate's problem to be quadratic, and thus can eliminate the difficulties of multiple optima. The contribution is that using Jennergren's scheme, coordination through pricing alone can allow each subordinate to find the optimal solution.

In presenting a limited number of algorithms as has been done, an attempt has been made to demonstrate that most computational solution procedures can be interpreted as an iterative exchange of information between the superordinate and the subordinates. The information communicated downward is in the form of a price for each coupling constraint. The information communicated upward is a solution, or some characteristics of a solution such as the profit or amount of organizational resources used by the solution. Table 1 summarizes the relevant features of the algorithms which have been discussed.

Conclusions

This chapter has attempted to present a thorough discussion and synthesis for coordination through goal intervention when the superordinate's objective function is equal to the sum of the subordinates' objective functions. These coordination mechanisms are called pricing approaches because they can be interpreted as assigning prices to constraints which couple the subordinates together. It was shown how the pricing schemes are derived from the general model given in Chapter II. The existence conditions for coordinating prices were stated, and several algorithms were discussed for conceptualizing the information exchange between superordinate and subordinates.

Historically, economists have applied pricing approaches to model decentralization in a competitive economic system. The value of

Table 1. Summary of Some Representative Pricing Algorithms.

Type of Model	Algorithm	Features	Complications
Linear	Dantzig-Wolfe (1958) [37,38]	1. Primal feasible after a certain point. 2. Converges finitely	1. Behavioral Disadvantage: Superordinate must make final decision for subordinates.
	Balas (1966) [15]	1. Prices are based on amount of infeasibility in primal problem. 2. Finite Convergence. 3. Dual feasible (but not primal feasible until final iteration).	1. Behavioral Disadvantage: Superordinate must make final decision for subordinates. 2. The superordinate must know the subordinates basis matrix at each iteration.
	Jennergren (1972) [60,61]	1. Uses a price schedule for each subordinate instead of a single price. 2. Converges finitely. 3. Primal feasible. 4. Subordinate's objective function is quadratic.	1. The optimal dual multipliers must be known.
Quadratic (concave objective, linear constraints)	Hass (1967) [56]	1. Can handle simple externalities in the objective function. 2. Primal feasible. 3. If objective function has no externalities and is strictly concave, then the optimal prices can be found.	1. Not necessarily finite. 2. With externalities the superordinate must make final decision for subordinates.

Table 1. (Continued)

Type of Model	Algorithm	Features	Complications
Quadratic (concave objective, linear constraints)	Whinston (1962) [119]		
Non-Linear (Strictly concave differentiable objective function, convex differentiable constraints)	Lasdon and Schoeffler (1966) [73] Uzawa (1958) [114] Lasdon [72]		1. Dual Feasible (but not primal feasible). 2. Not necessarily finite.

such models in the context of decentralized hierarchical resource allocation decision making processes is limited by such properties as:

(1) Most pricing approaches are not primal feasible (with some exceptions, e.g., Dantzig-Wolfe) which means that if the iterative process of information exchange between superordinate and subordinates terminates before an optimal solution has been found, then the current plan of resource allocations may be infeasible with respect to the coupling constraints.

(2) The conditions under which one can guarantee that there exists a set of prices which when communicated to the subordinates will allow them to find an overall optimum solution without any additional information, are quite restrictive and difficult to check, i.e., the objective function must be strictly concave, and the constraints must be convex and satisfy a constraint qualification. Several methods including the Dantzig-Wolfe procedure are often called pricing approaches, but they require that the superordinate communicate additional information to the subordinates other than just prices.

(3) The author has yet to find a description of a "real world" organization which uses a pricing approach for coordinating resource allocation decisions except under very special circumstances.[15]

The conclusions are summarized in the following proposition.

<u>Proposition 1</u>

A hierarchical decentralized (cooperative) organization which <u>uses a pricing procedure</u> to coordinate resource allocation decisions may

[15] These special circumstances refer to the use of transfer pricing which applies only when a multi-division organization sells products between divisions.

not arrive at an optimal solution with respect to the overall utility function if any of the following conditions are present

(1) there exist nonlinear dependencies (externalities) between decision making units at the same level;

(2) the overall utility function does not display strict decreasing returns to scale; or

(3) the constraint functions are not convex.

CHAPTER IV

COORDINATION THROUGH CONSTRAINT INTERVENTION IN A COOPERATIVE ORGANIZATION: RESOURCE BUDGETING APPROACHES

This chapter presents and analyzes models of constraint intervention when the superordinate's objective function is equal to the sum of the subordinates' objective functions. Such techniques are often called resource budgeting methods because the superordinate coordinates the activities of the subordinates by specifying how much of each organizational resource is available for a subordinate[16]. Thus, in a resource budgeting method the superordinate sets a resource budget for each subordinate.

There is a direct relationship between resource budgeting and pricing approaches [46, p. 376]. The distinction between the two amounts to resource budgeting approaches being primal methods while pricing methods are usually dual methods. As it will be shown it is quite easy to guarantee that throughout a resource budgeting coordination process, a feasible solution to the organization's problem is maintained.

In the opinion of the author resource budgeting coordination mechanisms are used in the budgeting process of many organizations and economic systems. For example, Kornai and Liptak describe the current

[16] In the literature of mathematical programming the resource budgeting methods are often called resource directive approaches [46, 102] and coordination through direct means [96, 113]. These terms are purposely avoided to minimize possible misinterpretations and to emphasize interest with the conceptual decentralized allocation process.

planning practice for a socialist economic system in the following way:[17]

> "The National Planning Bureau, acting on the basis of the requirements of economic policies and of general information about the various sectors, works out a preliminary draft plan which contains general targets (quota figures) for the sectors. The centre makes a provisional distribution of the available resources, material, manpower, etc. among the sectors, and at the same time also allocates their output targets. The sectors then proceed, through their own detailed calculations made on the basis of their concrete conditions, to give "substance" to the quotas and to lend concrete meaning to the central targets. In so doing, they also make recommendations for changes to the Planning Bureau. This is what is in economic usage called "counter-planning". On the basis of the counter-plans the National Planning Bureau modifies its original targets and again sends them down to the sectors. The method proposed here is an attempt to aid this process of planning and counter-planning by means of objective criteria.
>
> The procedure recommended also simulates the usual practice of planning in another respect. It repeatedly happens that the centre gives the sectors certain directives and asks them to report on the degree of economic efficiency with which the task can be carried out. The sectors express the efficiency of their activities through various "indices of economic efficiency", whose structure is prescribed by the centre." [66, p. 143]

In many organizations resource allocation budgeting decisions are made via an iterative process whereby a superordinate sets tentative resource budgets for subordinates. Given a tentative budget a subordinate decision unit decides how it would use the budget and what value would result from its plan. Using this information the superordinate may adjust the budget and the process starts over. This simplified description represents the major aspects of how a resource budgeting approach works.

In this chapter the resource budgeting mechanism will be derived from the general model given in chapter II. Next, the existence

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[17]Recall that an organization is viewed as a goal seeking system, and hence an economic system can be thought of as an organization [78].

conditions under which resource budgeting techniques coordinate in a cooperative organization are stated. Using a framework suggested by Geoffrion [46] three basic strategies for the development of resource directive algorithms (an algorithm can be used to describe the coordination through budgeting process) are discussed from the viewpoint of what information must be communicated between superordinate and the subordinates. As Geoffrion states,

> "The economic interpretations and implications of information flows and the resource directive and price directive mechanisms are quite interesting from the point of view of the decentralized organization. Many aspects of the resource directive approaches mentioned here have yet to be systematically studied from the economic viewpoint." [46, p. 400]

The main result in this chapter is to study and discuss the economic and behavioral interpretations and implications of coordination via a resource budgeting method for each of Geoffrion's three strategies.

Derivation of the Resource Budgeting Mechanism

Given that the superordinate's objective function is equal to the sum of the subordinate's objective functions, the general model expressed in (2-1) to (2-6) can be written as a single optimization problem:

Problem R

$$\text{maximize} \quad \sum_{i=1}^{n} f_i(\underline{x}_i)$$

$$\text{subject to:} \quad \sum_{i=1}^{n} \underline{\alpha}_i \leq \underline{b} \tag{4-1}$$

$$\sum_{i=1}^{n} \underline{\Psi}_i \, (\underline{\alpha}_i) \leq \underline{n} \tag{4-2}$$

$$\underline{h}_i \, (\underline{x}_i) \leq \underline{\alpha}_i \qquad \text{for } i = 1, \ . \ . \ . \ , n \tag{4-3}$$

$$\underline{x}_i \ \varepsilon \ X_i$$

Problem R can be partitioned up into n parts and the resulting decision problem for subordinate i is:

<u>Problem $R_i(\alpha_i)$</u>

maximize $\quad f_i \, (\underline{x}_i)$

Subject to: $\quad \underline{h}_i \, (\underline{x}_i) \leq \underline{\alpha}_i \qquad$ (4-4)

$$\underline{x}_i \ \varepsilon \ X_i$$

The superordinate's task is to iteratively choose $\underline{\alpha}_i$ for i = 1, . . . , n, such that

$$\sum_{i=1}^{n} \underline{\alpha}_i \leq \underline{b}$$

$$\sum_{i=1}^{n} \underline{\Psi}_i(\underline{\alpha}_i) \leq \underline{\eta}$$

until an optimal solution is found.

An important distinction should be made concerning problem R and the overall problems studied by others [46, 72, 102, 113]. The constraint (4-2) is not included in these other studies. In this chapter the $\underline{\Psi}_i(\underline{\alpha}_i)$ functions are not divided up and assigned to subordinate i as they would be in the other studies. Here the $\underline{\Psi}_i(\underline{\alpha}_i)$ functions remain as part of the constraints in the superordinate's problem. Hence, it is not required that the subordinates know or be concerned with constraint (4-2). In addition, even though (4-2) is written in a separable way, it is not necessary that it be separable. Therefore, (4-2) could be given by $\underline{\Psi}(\underline{\alpha}_1, \ldots, \underline{\alpha}_n) \leq \underline{\eta}$ and the theorems which are developed later still hold.

If one considers problem R as the resource allocation problem facing a two level profit maximizing organization, the resource budgeting coordination mechanism can be interpreted in the following way. The superordinate allocates a vector of resources, $\underline{\alpha}_i$, to subordinate i. The selection of $\underline{\alpha}_i$ is performed so that $\sum_{i=1}^{n} \underline{\alpha}_i \leq \underline{b}$ and $\sum_{i=1}^{n} \underline{\Psi}_i(\underline{\alpha}_i) \leq \underline{\eta}$. Upon receiving the vector of resources, $\underline{\alpha}_i$, subordinate i solves his decision problem, $R_i(\underline{\alpha}_i)$. This is much like the subordinate getting a tentative budget to be used in carrying on his activities. The solution to $R_i(\underline{\alpha}_i)$, $\underline{x}_i^*$, represents a potential profit to the organization of $f_i(\underline{x}_i^*)$. Each subordinate communicates to the superordinate his contribution to organizational profit and some summary information about his his profit would change if changes were made in the components of $\underline{\alpha}_i$. Using this information the superordinate determined if a choice of a new set of $\underline{\alpha}_i$'s could improve the total profit, if such a set of $\underline{\alpha}_i$'s exists he communicates to the superordinate his contribution to organizational profit and some summary information about how his profit would change if changes were made in the components of $\underline{\alpha}_i$. Using this information the superordinate determines if a choice of a new set of $\underline{\alpha}_i$'s could improve the total profit. If such a set of $\underline{\alpha}_i$'s exists he communicates them to the subordinates who resolve their problem, etc. The process iterates until the superordinate can find no way to further improve total profit.

All three strategies for algorithms suggested by Geoffrion can be interpreted in the above manner. However, from the viewpoint of a decentralized organization, all three strategies differ in the type and amount of information which a subordinate must communicate to the superordinate. It is important to understand if these three strategies make assumptions about what information must be communicated which is unrealistic from a behavioral viewpoint.

The next two sections will address the following two questions about the iterative process described above: (1) Under what conditions does there exist a set of $\underline{\alpha}_i$'s which when assigned to the subordinates generate an overall optimal solution? (2) Given this optimal set of $\underline{\alpha}_i$'s exist, what algorithms are available for finding them?

Existence Conditions

No as No assumptions are required in order to prove that a set of coordinating allocations exist as the following theorem demonstrates

Theorem 4.1

Suppose there exists an optimal solution $\underline{x}_i^*$ $(i = 1, \ldots, n)$ to problem R, then there exists a set of $\underline{\alpha}_i^*$ $(i = 1, \ldots, n)$ which when allocated to subordinate i, generates an optimal solution to problem R.

Proof:

Given $\underline{x}_i^*$, set $\underline{\alpha}_i^* = \underline{h}_i(\underline{x}_i^*)$ and solve problem $R^i(\underline{\alpha}_i^*)$ to get $\overline{\underline{x}}_i$. Thus, $f_i(\overline{\underline{x}}_i) \geq f_i(\underline{x}_i)$ for all $\underline{x}_i \in X_i$ and $\sum_{i=1}^{n} f_i(\overline{\underline{x}}_i) \geq \sum_{i=1}^{n} f_i(\underline{x}_i)$.

Therefore, since $\underline{x}_i^*$ was optimal, it must be true that

$$\sum_{i=1}^{n} f_i(\overline{\underline{x}}_i) = \sum_{i=1}^{n} f_i(\underline{x}_i^*).$$

In spite of the simplicity of this result, the ramifications of Theorem 4.1 are remarkable. It means that a resource budgeting approach can work under any circumstances. Unfortunately, the procedure for finding the optimal α_i's by the iterative interplay between superordinate and subordinate cannot be guaranteed to work under all conditions.

Recently, ten Kate [111] has proposed necessary conditions for an optimal solution when using a resource budgeting procedure.

[18] Both Silverman [102, p. 61] and Geoffrion [40, p. 380] state a somewhat similar result. However, their statements are in a slightly different context.

The conditions which are stated in his "direct distribution" theorem are theoretically valid, but of little use from an applications viewpoint. In order to indicate why, his result is given in Theorem 4.2.

Theorem 4.2 (ten Kate [113])

Suppose problem R satisfies a constraint qualification and the constraints represent in (4-2) are not present. If $f_i(\underline{x}_i)$ and $\underline{h}_i(\underline{x}_i)$ are differentiable and $\underline{x}_i$*, $\underline{\alpha}_i$* is optimal for problem R, then $\underline{\pi}_i$*, the vector of optimal dual variables for (4-3), is also an optimal dual variable for (4-4) in problem $R_i(\alpha_i^*)$.

The proof as given by ten Kate [113, p. 737] depends on the Kuhn Tucker stationary point conditions. The significance of the theorem would seen to be that if during the iterative exchange between superordinate and subordinates, a solution is generated by the subordinates, and the shadow prices of each subordinate are not equal, then the superordinate knows that this solution is not optimal. Unfortunately, in terms of practical application this theorem is of little value for three reasons:

(1) Having all the shadow prices for the subordinates equal is a necessary but not sufficient condition for overall optimality, i.e., if the superordinate chooses this condition as a stopping rule for the iterative process he may not get an optimal solution, e.g., tenKate [113, p. 740] gives a numerical example.

(2) If the subordinate's problem has multiple dual optimal solutions, i.e., the optimal problem solution is degenerate, then even though overall optimality is present the shadow prices for each subordinate may be unequal.

Example 4.1

Suppose the overall problem is given by:

$$\text{maximize} \quad 10X_{11} + 14X_{12} + 19X_{21} + 13X_{22}$$

$$\text{subject to:} \quad 2X_{11} + 3X_{12} \le \alpha_1$$

$$4X_{21} + 3X_{22} \le \alpha_2$$

$$\alpha_1 + \alpha_2 \le 7$$

$$0 \le X_{11}, X_{12}, X_{21}, X_{22} \le 1$$

The optimal solution for this example is

$$\alpha_1^* = 3, \; X_{11}^* = 1, \; X_{12}^* = 1/3$$

$$\alpha_2^* = 4, \; X_{21}^* = 1, \; X_{22}^* = 0.$$

Using the α_i^* above the subordinate's problems are:

Subordinate 1

$$\text{maximize} \quad 10X_{11} + 14X_{12}$$

subject to: $2X_{11} + 3X_{12} \leq 3 \quad (\pi_1)$

$$0 \leq X_{11}, X_{12} \leq 1$$

Subordinate 2

maximize $19X_{21} + 13X_{22}$

subject to: $4X_{21} + 3X_{22} \leq 4 \quad (\pi_2)$

$$0 \leq X_{21}, X_{22} \leq 1$$

The solution to the subordinates problems yields the correct solution. However, the dual of subordinate 2 has multiple optimal solutions[19]. The extreme point dual optima are $\pi_2^1 = 13/3$ and $\pi_2^2 = 19/4$. The dual variable for subordinate 1 is $\pi_1 = 14/3$. Clearly, $\pi_1 \neq \pi_2$ unless one selects a particular convex combination of π_2^1 and π_2^2. In particular, if $w = 1/5$, then $\pi_2 = w\pi_2^1 + (1-w)\,\pi_2^2 = 14/3$, but during the solution processit is unlikely that the subordinate would report such a non extreme point of value.

(3) If constraints like (4-2) are present, then tenKate's theorem does not hold. Intuitively, the reason is that restrictions

[19]The multiplicity of dual solutions indicates that the function $v_i(\underline{\alpha}_i)$ $= \text{maximum } f_i(\underline{x}_i)$ Subject to: $h_i(\underline{x}_i) \leq \underline{\alpha}_i$

$$\underline{x}_i \in X_i$$

is not differentiable at that value of $\underline{\alpha}_i$ which caused the multiple dual solutions. [72, p. 471] like (4-2) are not portitioned up and assigned to the subordinates as with (4-1). Therefore, whenever there are restrictions on the manner in which a budget can be allocated between subordinates, e.g., if the defense budget cannot exceed the health, education, and welfare budget, then overall optimality is not necessarily confirmed or denied if the change in value per unit change in budget for every subordinate is equivalent.

To summarize, the existence conditions for resource directive coordination are very general. However, without making additional assumptions it is very difficult to ascertain when an optimal solution has been reached.

Resource Budgeting Algorithms

The section summarizes the key points about the three strategies for solving problem R through resource budgeting. It is necessary to give a brief mathematical explanation of each strategy so that the behavorial implications can be analyzed. The reader who wants more mathematical details should see Geoffrion [46] or Lasdon [72].

Through the use of a mechanism known as projection [20] problem R can be shown to yield the following equivalent problem:

Problem R'

$$\text{maximize} \quad \sum_{i=1}^{n} v_i(\underline{\alpha}_i)$$

---

[20]For a rigorous discussion of this mechanism see Geoffrion [47].

subject to: $$\sum_{i=1}^{n} \underline{\alpha}_i \leq \underline{b}$$

$$\sum_{i=1}^{n} \underline{\Psi}_i(\underline{\alpha}_i) \leq \underline{\eta}$$

$$\alpha_i \varepsilon \Gamma_i = \{ \underline{\alpha}_i \ \varepsilon \ E^m \ | \underline{h}_i(\underline{x}_i) \leq \underline{\alpha}_i \text{ for some } x_i \ \varepsilon \ X_i \} \quad (4\text{-}5)$$

where $v_i(\underline{\alpha}_i)$ is the supremal value of problem $R_i(\alpha_i)$, i.e.,

$$v_i(\underline{\alpha}_i) = \sup \left\{ \begin{array}{l} f_i(\underline{x}_i) \\ \text{s.t. } \underline{h}_i(\underline{x}_i) \leq \underline{\alpha}_i, \ \underline{x}_i \varepsilon \ X_i \end{array} \right\}$$

If $-f_i$, $\underline{h}_i$, and Ψ_i are convex functions on the convex set X_i, then it can be established that problem R' is a concave program and that Γ_i is a convex set [46, p. 380]. Therefore, in the context of a decentralized organization the superordinate's coordination problem can be represented by problem R', and subordinate i's problem can be expressed as problem $R_i(\alpha_i)$. However, solving problem R' presents some serious difficulties. First, $v_i(\underline{\alpha}_i)$, which is a characterization of subordinate i's optimal response as a function of his resource budget, is not known explicitly. Secondly, even if $v_i(\underline{\alpha}_i)$ is known, it is not differentiable, e.g., see Geoffrion [47], and thus gradient methods are not immediately applicable. Finally, without information about each subordinate's constraint set, the superordinate cannot guarantee that (4-5) is satisfied. To overcome these difficulties the strategies which Geoffrion chose to name as tangential approximation, large step subgradient and piecewise approaches can be utilized. Throughout the following discussion it is assumed that $-f_i$, $\underline{h}_i(\underline{x}_i)$, and $\underline{\Psi}_i$ are convex differentiable functions on the convex set X_i.

Tangential Approximation

Tangential approximation methods seek to build up a piecewise linear approximation or representation to $v_i(\underline{\alpha}_i)$ by evaluating the tangent of $v_i(\underline{\alpha}_i)$ at various points. If one assumes that the vector of dual variables, $\overline{\underline{\pi}}_i$, associated with (4-4) exists for a given $\overline{\underline{\alpha}}_i$, it can be shown that $\overline{\underline{\pi}}_i$ is the normal of the tangent to $v_i(\underline{\alpha}_i)$ at $\overline{\underline{\alpha}}_i$ [46, p. 381]. Given $\underline{\alpha}_i = \overline{\underline{\alpha}}_i$, the function

$$f_i(\overline{\underline{x}}_i) - \overline{\underline{\pi}}'_i(\overline{\underline{\alpha}}_i - \underline{\alpha}_i)$$

is a tangent to $v_i(\underline{\alpha}_i)$ at $\overline{\underline{\alpha}}_i$ where $\overline{\underline{x}}_i$ is the optimal solution to problem $R_i(\overline{\underline{\alpha}}_i)$. In fact with the additional assumptions that $f_i(\underline{x}_i)$ and $\underline{h}_i(\underline{x}_i)$ are upper semicontinuous functions and that X_i is a compact set, then

$$v_i(\underline{\alpha}_i) = \underset{\overline{\underline{\alpha}}_i \in \Gamma_i}{\text{minimum}} \left[v_i(\overline{\underline{\alpha}}_i) - \overline{\underline{\pi}}_i' (\overline{\underline{\alpha}}_i - \underline{\alpha}_i)\right]$$

for all $\underline{\alpha}_i \varepsilon \Gamma_i$ [46, p. 381].

These results suggest that problem R' can be solved by building up a better and better approximation of v_i in terms of its tangents. In Figure 4 v_i has been evaluated at three points: $\underline{\alpha}_i^{o}$, $\underline{\alpha}_i^{1}$, and $\underline{\alpha}_i^{2}$. (The first tangent has slope $\underline{\pi}_i$, the second $\underline{\pi}_i^{o}$, and the third $\underline{\pi}_i^{2}$.) The resulting approximation for $v_i(\underline{\alpha}_i)$ is the piecewise linear function formed by taking the minimum of the three supports. Using this approximation in place of v_i in problem R' might yield $\overline{\underline{\alpha}}_i$. The evaluation of v_i at $\overline{\underline{\alpha}}_i$ would result in another tangent whose use would improve the approximation. Thus, given enough iterations one can get as close as desired to the optimal solution.

Geofrion [46, p. 382] suggests the following solution procedure:

1. Starting with a set of $\underline{\alpha}_i^{o} \varepsilon \Gamma_i (i = 1, \ldots, n)$ which is feasible to the superordinate's constraints (4-1) and (4-2) subordinate i solves problem $R_i(\underline{\alpha}_i)$ to find $f_i(\underline{x}_i)$ and $\underline{\pi}_i$. Set $v = 0$.

2. Solve the following problem which is equivalent to the current tangential approximation of problem R'.

<u>Problem R"</u>

maximize
$$\sum_{i=1}^{n} \sigma_i^{v}$$

subject to:
$$\sigma_i \leq f_i(\underline{x}_i^{j}) - \underline{\pi}_i^{j\prime}(\underline{\alpha}_i^{j} - \underline{\alpha}_i)$$

$$j = 1, \ldots, v$$

$$i = 1, \ldots, n$$

$$\sum_{i=1}^{n} \underline{\alpha}_i \leq \underline{b}$$

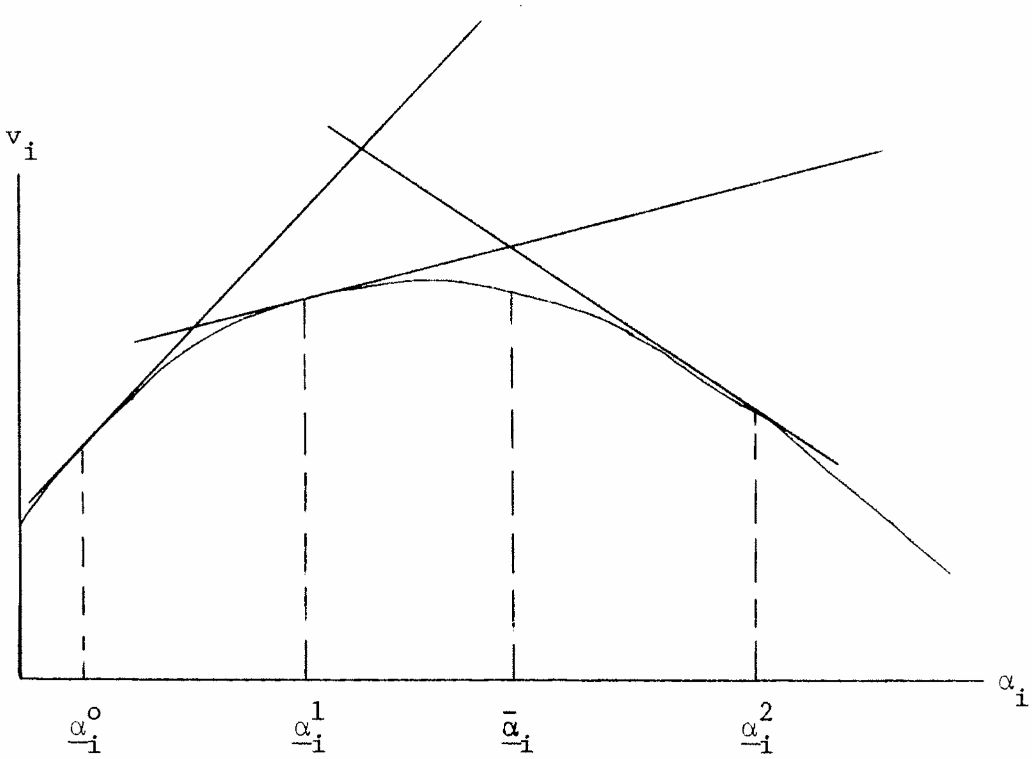

Figure 4. Construction of Piecewise Linear Approximation to v_i using Tangential Approximation.

$$\sum_{i=1}^{n} \underline{\Psi}_i(\underline{\alpha}_i) \leq \underline{n}$$

$$\underline{\alpha}_i \varepsilon \ \Gamma_i \qquad i = 1, \ldots, n.$$

Let $\underline{\alpha}_i^{v+1}$ be the optimal solution.

3. Subordinate i resolves problem $R_i(\underline{\alpha}_i^{v+1})$ to find $f_i(\underline{x}_i^{v+1})$ and $\underline{\pi}_i^{v+1}$.

4. If $\underline{\alpha}_i^{v+1}$ is sufficiently near optimal in problem R' terminate. Otherwise, increase v by 1 and return to step 2.

Geoffrion does not prove that such a procedure converges to an optimal solution but indicates that "each approach here should converge to an optimal solution, if one properly attains to the tactical questions." [46, p. 377] In Chapter VI of this dissertation a specific algorithm for conceptualizing a coordination procedure is presented and shown to converge finitely. This algorithm can be derived using Geoffrion's strategy although another development is given there.

Information concerning upper and lower bounds for the optimal objective function value is generated at each iteration of the above procedure. Geoffrion [46] shows that at iteration v of the tangential approximation procedure the value $\sum_{i=1}^{n} \sigma_i^{v*}$ (where σ_i^{v*} is optimal in problem R") is an upper bound on $\sum_{i=1}^{n} f_i(\underline{x}_i^*)$ the optimal objective function value of problem R. Furthermore, $\sum_{i=1}^{n} f_i(x_i\ )$ is a lower bound for $\sum_{i=1}^{n} f_i(\underline{x}_i^*)$. Therefore at each iteration the superordinate has some idea of how for from the optimum the current solution is. Another appealing feature of tangential approximation is that the solution,

$\underline{x}_i^v (i = 1, \ldots, n)$ at each iteration, v, is feasible in terms of problem R. Therefore, if the solution algorithm is terminated before an optimal solution is found, at least the organization has a solution which can be implemented and an indication of how far that solution is from being optimal.

To the author's knowledge no economic interpretation has appeared for the tangential approximation procedure. However, there is a very straightforward (and probably well known to those in mathematical programming) and appealing interpretation available. In step 1 the superordinate is establishing a preliminary tentative resource budget for each subordinate i who decides on a resource allocation plan. Subordinate i then informs the superordinate of the maximum value he could attain for that budget. In addition, he informs the superordinate of how this value might change if his resource budget were increased or decreased. The information communicated to the superordinate corresponds to $f_i(\underline{x}_i)$ and $\underline{\alpha}_i$. The superordinate uses this information to determine if a reapportionment of the resource budget could bring about an overall increase in value. If so, he communicates the new budget and the process continues. In terms of information flows and the iterative nature of the process, the above procedure seems descriptive for many organizations. Of course, in actual application the procedure would iterate only a few times.

Computationally, the greatest difficulty of the tangential approximation mehtod is in handling the restriction expressed in (4-5). In the context of a decentralized organization this means that the superordinate must never assign a resource budget to a subordinate for which he has no feasible solution. Explicitly, such a restriction implies that the superordinate knows Γ_i, i.e., he knows the constraint set of each subordinate. This violates the informational automony assumption. In certain situations the superordinate need not concern himself with Γ_i. For example, if subordinate i's problem possesses a feasible solution for any non-zero resource budget, then $\Gamma_i = \{\underline{\alpha}_i \in E^m | \underline{\alpha}_i \geq \underline{0}\}$. Often, this is a satisfactory assumption because if subordinate i is allocated none of a particular resource, then projects which require this resource cannot be undertaken, and thus no value is obtained from the project. In cases, where subordinate i must support a certain project, then if subordinate i can communicate to the superordinate, the minimum resource budget for which he can attain a feasible solution, then the superordinate can explicitly set $\Gamma_i = \{\underline{\alpha}_i \in E^m, \underline{\alpha}_i \geq \underline{b}_i^{min}\}$ where $\underline{b}_i^{min}$ is the minimum budget. It is felt that for most organizations either of these two assumptions can be carried out.

If problem R is linear it is possible to generate Γ_i explicitly. Kohler [63] shows that Γ_i can be expressed as a finite collection of linear equalities. Kohler [63] and Zschau [127] have presented algorithms for generating Γ_i. In fact Zschau shows how Γ_i can be generated during the information exchange between superordinate and the subordinate. However, to accomplish this the subordinate must communicate all extreme points of his constraint set. Therefore, such a method involves a substantial increase in the amount of information which must be communicated, and as Geoffrion states, "the computational burden is likely to be excessive." [46, p. 385] Geoffrion suggests using relaxation [47] whereby one temporarily ignores all but a subset of the constraints defining each Γ_i. If a $\underline{\alpha}_i \notin \Gamma_i$ is generated then a constraint is generated which dissallows this. ten Kate [112] presents a formal method for accomplishing the relaxation strategy. The main difficulty with using a relaxation strategy is that there does not appear to be any meaningful interpretation associated with the information which must be communicated to generate the violated constraint.

Geoffrion [46] also suggests two schemes for generating an approximation or representation for Γ_i by:

(1) Building up an "adequate" outer (containing) polyhedral representation as needed based on supporting hyperplanes.

(2) Building up an adequate inner (contained) polyhedral representation as needed based on points in Γ_i.

However, both these schemes and the relaxation strategy discussed in the previous paragraph may be computationally plausible, but they have little value in conceptualizing a decentralized planning procedure because they either violate the informational autonomy assumption, or they require the transmission of information which has no meaningful behavorial interpretation.

Tangential approximation methods can be used to build up a piecewise linear representation of the overall objective function as a function of the amounts of resources allocated to the subordinates. For purposes of describing coordination through budgeting in a decentralized organization tangential approximation strategies are appealing only when Γ_i can be described easily. However, it many organizations this may be the case.

Large Step Subgradient Methods

Given an initial resource allocation for the subordinates which satisfies the constraints of problem R', large step subgradient approaches use gradient like procedures to determine a new set of feasible reallocations for the subordinates which improve the overall solution. The difficulty of this approach is that $v_i(\underline{\alpha}_i)$ is not differentiable every where, and thus a gradient approach cannot be used. However, a method which utilizes subgradients is applicable. Geoffrion suggests the following procedure:

(1) Let $\underline{\alpha}_i^o$, i=1, . . . , n, staisfy the constraints

$$\sum_{i=1}^{n} \underline{\alpha}_i^o \leq \underline{b} \text{ and } \sum_{i=1}^{n} \underline{\Psi}_i(\underline{\alpha}_i^o) \leq \underline{\eta}$$

Given $\underline{\alpha}_i$, subordinate i solves problem $R_i(\underline{\alpha}_i)$.

(2) Determine a "good" improving feasible direction z_i at $\underline{\alpha}_i$; if no improving feasible direction exists, then $\underline{\alpha}_i$ is optimal.

(3) Determine a step size θ in the direction $\underline{z}_i$ by solving the following problem:

$$\text{maximize} \quad \sum_{i=1}^{n} v_i(\underline{\alpha}_i^o + \theta \underline{z}_i^o)$$

$$\text{subject to:} \quad \sum_{i=1}^{n} (\underline{\alpha}_i^o + \theta\, \underline{z}_i^o) \leq \underline{b}$$

$$\sum_{i=1}^{n} \underline{\Psi}_i(\underline{\alpha}_i^o + \theta\, \underline{z}_i^o) \leq \underline{\eta}$$

until the formula changes.

Denote $\underline{\alpha}_i^o + \theta^o \underline{z}_i^o$ by $\underline{\alpha}_i^1$, and resolve problem $R_i(\underline{\alpha}_i^1)$. Return to step

(2) with $\underline{\alpha}_i^{\circ}$ replaced by $\underline{\alpha}_i'$.

The primary difficulty is how to carry out step (2). It can be shown [46, 102, 127] that the positive directional derivatives of $\sum_{i=1}^{n} v_i(\underline{\alpha}_i)$ determine the set of all feasible improving directions. The essential difference between various algorithms is the criteria used to select an improving direction. For example, Geoffrion [46] chooses an improving feasible direction which maximizes the initial rate of improvement. His direction finding problem can be written as a block diagonal linear programming problem [46, p. 394] which has coefficients representing the gradients of f_i , $\underline{h}_i$, $\underline{\Psi}_i$, and the gradients of the functions defining the set X_i evaluated at the most recent solution of problem $R_i(\underline{\alpha}_i)$.

The interpretation of such a procedure in a resource allocation process is that the superordinate fixes a resource budget for each subordinate. Each subordinate then solves its problem $R_i(\alpha_i)$ for the given budget and reports back the gradient of f_i, $\underline{h}_i$, $\underline{\ }_i$ and the gradient of functions defining the set X_i evaluated at the solution found to problem $R_i(\alpha_i)$. It is well known that a gradient represents the direction of the greatest local increase in a function evaluated at a given point. In the context of a decentralized organization there does not appear to be a suitable behavorial interpretation associated with the information which must be transmitted from subordinate to superordinate, i.e., the gradients of f_i, $\underline{h}_i$, $\underline{\psi}_i$, and the functions defining X_i evaluated at the current solution. The superordinate uses this information to determine a direction $\underline{z}_i$, i = 1, . . . , n to move in changing the previous allocations. Finally, if it is desired to find the optimal distance to move along the direction found, the problem in step (3) is solved. To solve this problem optimally it is necessary to parametrically solve each subordinate's problem, parameterizing over all feasible step sizes, θ [46, p. 390]. This can be accomplished by each subordinate reporting the maximum amount the current budget could be increased before the subordinate's objective function ceases to increase. It is not necessary to select the optimal step size, but only that the step size be selected so that

$$\sum_{i=1}^{n} (\underline{\alpha}_i^K + \theta \underline{z}_i^K) \leq \underline{b}$$

and

$$\sum_{i=1}^{n} \underline{\Psi} (\underline{\alpha}_i^K + \theta \underline{z}_i^K) \leq \underline{\eta}$$

However, if the optimal step size is not chosen, it would appear that the process would converge less rapidly than if it were.

After choice of a direction and a step size, a new set of budgets for the subordinates is chosen, and the process begins over. Thus, one can interpret the large step subgradient approach for resource allocation in a decentralized hierarchical organization as a process by which tentative budgets are iteratively set for subordinates by using certain information supplied by the subordinates. However, in terms of a decentralized organization the information which is communicated upward does not seem to be a realistic description for the kind of information flow which might actually take place. Also, when choosing the step size it is necessary to know how large θ can be made before $(\underline{\alpha}_i^K + \theta \underline{z}_i^K) \notin \Gamma_i$. This requires additional interactions between the superordinate and the subordinates. Therefore, large step subgradient methods must be considered primarily as computational devices, and not as a descriptive model of the budgeting process in a decentralized hierarchical organization.

Examples of algorithms based on the large step subgradient approach are given by Brosilow, Lasdon, and Pearson [23], Abadie and Sakorvitch [1], Zschau [127] and Silverman [102].[21] With the exception of Zschau's method, these algorithms were developed as a way to solve large mathematical programming problems, and thus their originators were not concerned with any economic interpretations. Zschau's algorithm "did not arise out of algebraic concepts, nor was it developed primarily as a computational tool. Rather it was developed as a way of formalizing an apparently common decision making procedure for decentralized organizations," [127, p. 1]. Zschau's procedure is confined to a linear problem with the result being that the procedure given earlier can be simplified. An important simplification comes in step (2) where the superordinate no longer must find the "best" direction for a reallocation. In effect, the subordinate no longer need communicate the gradients of the functions in his problem but instead reports how much he would be willing to pay or how much he would be willing to sell a portion of his resources for. This amount depends upon the change in the subordinate's objective function. Specifically, the subordinate must communicate all "basic" proposals (extreme points

[21]Geoffrion [48] and Geoffrion and Hogan [50] have also presented a subgradient approach for the general model given in (2-1) to (2-6) with the assumption that the superordinate's objective function is a concave funtion of $v_i(\underline{\alpha}_i)$, the subordinate's optimal response.

of his constraint set) for each tentative resource budget set by the superordinate. The superordinate uses this information to arrive at a new allocation [127, p. 63]. Zschau's algorithm converges finitely but appears to be computationally inefficient [60, p. 52].

Piecewise Approach

This strategy attempts to use the observation that the functions v_i have a significantly simpler structure over certain regions of E^m because different subsets of the constraints in problem $R_i(\underline{\alpha}_i)$ are active for different values of $\underline{\alpha}_i$. Thus, if one can subdivide E^m into regions for which v_i has a relatively simple structure and then seek an optimal solution to problem R' in piecemeal fashion each time restricting $\underline{\alpha}_i$ to one of the regions, the problem can be simplified. This approach has been developed for linear and quadratic problems. The procedure given below is illustrative of a linear problem.

(1) Starting with a $\underline{\alpha}_i{}^o$ which satisfies

$$\sum_{i=1}^{n} \underline{\alpha}_i{}^o \leq \underline{b} \text{ and } \sum_{i=1}^{n} \underline{\Psi}_i(\underline{\alpha}_i{}^o) \leq \underline{n}$$

problem $R_i(\alpha_i{}^o)$ is solved. Denote by $\overline{R}_i{}^o$ the region (convex polyhedral) of E^m on which the current optimal basis for $R_i(\underline{\alpha}_i{}^o)$ remains optimal, and let $\underline{w}_i{}^{o\prime}\underline{\alpha}_i{}^o$ be the value of v_i on this region.

(2) Solve problem R' with $\Gamma_i = \{ \underline{\alpha}_i \varepsilon E^m | \underline{\alpha}_i \varepsilon \overline{R}_i{}^o \}$. Let $\underline{\alpha}_i{}'$, $i = 1, \ldots, n$ be an optimal solution.

(3) If $\underline{\alpha}'$ is optimal in problem R', terminate. Otherwise, identify and change to an alternate optimal basis in some of the subproblems, $R_i(\underline{\alpha}_i')$, so that $\underline{\alpha}_i{}'$ is free to move in an improving

feasible direction for problem R' without requiring a further basis change in any of the subproblems. Determine the corresponding new regions $\bar{R}_i$ and functions $\underline{w}_i'$ and return to step (2) with α_i^o, $\bar{R}_i^o$ and $\underline{w}_i^o$ replaced by $\underline{\alpha}_i'$, $\bar{\underline{R}}_i'$, and $\underline{w}_i'$.

For a linear problem, the set $\bar{R}i$ can be identified. For example, if B^{-1} is the optimal basis inverse and $\underline{q}$ is the original right side except the numberical values which correspond to $\underline{\alpha}_i$ are replaced by the unknowns $\underline{\alpha}_i$, then Γ_i is given by $\Gamma_i = \{ \underline{\alpha}_i \varepsilon E^m \mid B^{-1}\underline{\alpha}_i \geq 0 \}$. In a similar way it is easy to get an expression for $\underline{w}_i$ in terms of $\underline{\alpha}_i$ by using the dual objective function.

In step 3, to ascertain whether a particular $\underline{\alpha}_i$ is optimal in problem R' it is necessary to determine whether an improving direction exists. This determination can be made by solving the direction finding problem given in the previous section (which means the subordinate must communicate his constraint matrix to the superordinate) or by making use of the theory of linear programming [46, p. 396]. In either case, if an improving direction is found, the superordinate must cause at least one subordinate to find an alternative optimal dual solution. This results in the construction of a new region $\bar{R}_i$ and a new function $\underline{w}_i$.

The economic interpretation of this approach is that as in all resource budgeting methods, the superordinate iteratively communicates tentative resource budgets to each subordinate. Each subordinate solves his problem and communicates to the superordinate the value of the optimal dual variable associated with the organizational resource constraints, and the set $\bar{R}_i$ which defines the ranges for the resource budgets over which the current basis does not change. In a sense this set defines the maximum and minimum amounts of resources for which the set of currently funded projects will be undertaken. The dual multipliers demonstrate how the subordinate's objective function will change over this set. The superordinate uses this information to determine if a new budget sould improve the overall objective function. If one exists, he communicates it to certain subordinates. These are the subordinates whose new $\underline{\alpha}_i$ is on the boundary of the set $\bar{R}_i$. For these subordinates there will be multiple dual solutions. The subordinate communicates one of the dual solutions which is different from the previous one communicated. The superordinates uses this information to adjust the budgets and the process begins anew.

Examples of algorithms using the piecewise approach include Rosen [89] and Varaiya [115]. Geoffrion shows how the piecewise approach could be used for linear and quadratic problems, however, it would appear that for general nonlinear problems the piecewise approach cannot be used because there is no easy way to subdivide E^m into regions over which v_i has a simple structure.

Other Approaches

Kornai and Liptak [66] and Weitzman [118] have proposed resource budgeting approaches for the problem of economic planning for a socialist country from a decentralized viewpoint. However, theor procedures have some relevance to resource allocation in a hierarchical decentralized organization.

Kornai and Liptak [66] appear to have been the first to propose a budgeting approach for decentralized planning. Their method is much like tangential approximation. Their algorithm for describing the coordinating procedure for a linear problem where the divisional restrictions $\underline{x}_i \varepsilon X_i$ are not present assumes that the superordinate's objective function can be approximated by:

$$\text{maximize} \quad \sum_{i=1}^{n} v_i(\underline{\alpha}_i) = \max \sum_{K=1}^{n} \min \hat{\underline{\pi}}_K{}'\underline{\alpha}_K.$$

The difficulty of their approach is that the optimal dual multiplier, $\hat{\underline{\Pi}}_K$, is only valid for a certain range of $\underline{\alpha}_i$ values. The range of $\underline{\alpha}_i$ can be divided into a number of regions each having its own dual variable, but the superordinate knows only $\hat{\underline{\pi}}_K$ which is valid only in a neighborhood of $\underline{\alpha}_i$. To alleviate this difficulty, Kornai and Liptak use an averaging procedure for computing new budgets, $\underline{\alpha}_i$, which converges, but not finitely. In addition, ten Kate says that in practice the convergence is very slow [112, p. 2].

Weitzman's approach is much like the tangential approximation strategy where an outer approximation to Γ_i is developed, however, a different economic interpretation can be associated with the algorithm which is somewhat like the goal partitioning algorithm developed in Chapter VI.

In Weitzman's algroithm it is assumed that only the superordinate knows the overall objective function of the organization, and therefore, this function need not be separable in terms of the subordinates. Each subordinate (or sector of the economy) has a set Y_k which represents the production and resource capabilities and limitations. It is assumed that the superordinate has an estimate (possibly acquired from previous experience) of Y_k, say $\tilde{Y}_k$ with $\tilde{Y}_k \subseteq Y_k$, i.e., the superordinate has an overly optimistic idea of the production possibilities. Using $\tilde{Y}_k$ the superordinate maximizes his utility function and arrives at a tentative budget and performance goals for each subordinate. Subordinate k attempts to find a feasible allocation of its budgeted resources to activities so as to satisfy the performance goals set by the superordinate. Thus, the subordinate does not maximize but instead tries only to find a feasible solution. However, since the superordinate's estimate of Y_k was overly optimistic, it is likely that the subordinate has no feasible solution. The subordinate then communicates information which will cause the superordinate to out off part of Y_k so that its new estimate, $\tilde{\tilde{Y}}_k$ is such that $\tilde{Y}_k \supseteq \tilde{\tilde{Y}}_k \supseteq Y_k$. This is done by informing the superordinate of the characteristics of a plan which is feasible in regards to available resources and comes as close as possible "to achieving the performance goals that had been assigned. Weitzman shows how this information can be used to construct a hyperplane which cuts off part of the superordinate's estimate" of $\tilde{Y}_k$ [118, p. 54].

Based on the latest estimate of the subordinate's feasible region, the superordinate chooses new performance goals and resource budgets for the subordinates, and the process continues until a feasible solution is found by each subordinate. Under certain restrictions such as convexity and compactness, this approach can be proven to converge to an optimal solution. In practice, the superordinate could call a halt to the iterative procedure whenever performance goals are not too tight or too loose.

One should note the slight generalization with respect to interpretation that results from Kornai and Liptak and Weitzman's work. In addition to setting resource budgets, the possibility of specifying performance targets is also suggested. This is very similar to the technique suggested in Chapter VI. However, Kornai and Liptak and Weitzman assume that the organization is cooperative, and thus subordinates honestly try to meet the targets (both resource and performance) which have been set by the superordinate.

In the literature of systems theory Mesarovic, Macko and Takahara [84] have proposed the "interaction prediction principle" which appears to be a resource directive technique. Their principle allows the superordinate to predict what the interaction between subordinates will be, i.e., the superordinate predicts what each subordinate's resource budget will be. As Mesarovic, et al. state, "The principle simply states that the overall decision problem is solved . . . whenever infimal decision problems are solved by x and the interactions are correctly predicted". [84, p. 99] It would seem that the definition of interaction balance principle is just a statement Theroem 4.1. However, Mesarovic et al. do not propose any algorithms for finding the correct interactions.

Summary of Algorithms

The basis mathematical logic underlying the three main strategies for resource budgeting methods has been explained. Table 2 summarizes these strategies. An economic interpretation of the information communicated between superordinate and subordinates has also been associated

Table 2. Main Approaches for Solving Problem R.

Approach	Complications	Algorithms using this App.
1. Tangential Approximation: Problem R is solved by iteratively building up a tangential approximation for the objective function of R. The tangential approximation is automatically available from the optimal multipliers of problem $R_i(\alpha_i)$ for a given $\underline{\alpha}_i$.	1. It is sometimes hard to find feasible α_i's because of (4-5). 2. Most methods of handling (4-5) would require the superordinate to know something about the constraint sets of the subordinates.	Kornai and Liptak [66] Benders [21] Geoffrion [51] tenKate [112]
2. Large Step Subgradient: Because of the non-differentiability of Problem R's objective function, large step gradient algorithms cannot be used. However, the optimal multipliers of $R_i(\alpha_i)$ can be used to characterize v_i via subgradients, thereby enabling an optimal or near optimal choice of an improving $\underline{\alpha}_i$.	1. Algorithms for the non-linear case require the superordinate to know something about the constraints of the subordinates.	Zschau [127] Abadie and Sakarovitch [1] Silverman [102] Geoffrion [46] Brosilow, Lasdon, and Pearson [23]
3. Piecewise Approach: This approach capitalizes on the simple structure of v_i over certain regions of E^m. It attempts to subdivide E^m into regions on which the v_i have a relatively simple structure and then solve Problem R' in piecemeal fashion, each time with α_i restricted to one of these regions.	1. For any problems which are not linear or quadratic, this approach becomes unwieldly. 2. The subordinates transmit its current solution and information about how it would change over a certain set.	Rosen [79] Varaiya [115] Geoffrion [46]

with each of the three strategies. In all three strategies the superordinate iteratively sets budgets for the subordinates by solving an optimization problem. The problem solved is different depending on the strategy used. With all three strategies each subordinate decides what activities are to be supported by solving problem $R_i(\alpha_i)$. The primary distinction between the strategies concerns the nature of the information which is communicated by a subordinate to a superordinate. Tangential approximation requires the communication of the least amount of information, in that only the subordinate's objective function value and the vector of shadow prices, $\underline{\pi}_i$, which indicates the change in the objective function value as the resource budget is changed, is transmitted. The piecewise approach requires transmission of shadow prices and an indication of over what range of resource budgets, the subordinate's current set of chosen activities would not change. The subgradient method requires that the subordinate communicate the gradient of its objective function and all the constraint function gradients including those constraints common only to the subordinate. Because of the behavorial interpretations associated with the subgradient approach, it would seem to have little intuitive appeal as a mechanism for explaining what goes on during the resource allocation process.

From the viewpoint of descriptive value the tangential approximation procedure and the piecewise procedure are more appealing. Both would seem to have merit in certain environments. Empirical support for this assertion may be found in the description of the resource allocation process in Baker et al. [14], Shumway et al. [107], Kornai and Liptak [66], Weitzman [118, and Wildavsky [113].

From a computational standpoint as Geoffrion states, "the central unresolved issue is the relative efficiency of the three methods". [46, p. 400] He suggests that tangential approximation may be the best when the Γ_i sets are simple, but to date there is no substantiating test results.

Summary

This chapter has shown that the problem represented in Chapter II, (2-1) to (2-6), can be expressed as two related problems, problem R' and problem $R_i(\alpha_i)$, when the assumption is made that the superordinate's objective function is equal to the sum of the subordinates' objective functions. The solution to these two problems is accomplished through an iterative exchange of information between the superordinate and the subordinate by which the superordinate learns how the value of each subordinate's objective function changes as a function of the resources which are allocated to him.

It was shown that the conditions under which the superordinate can coordinate the activities of the subordinates by allocating resources to them for their use are very general. However, when trying to show that an algorithm can be used to iteratively find the optimal resource budget for the subordinates, it is necessary to make assumptions about the forms of the functions. The three general strategies as developed by Geoffrion for coordination through resource budgeting were presented. No general stopping rules for the strategies were given, but Grinold [54] does discuss this subject.

The economic interpretation of all resource budgeting methods is that the superordinate iteratively communicates a resource budget for each subordinate. Each subordinate decides how to use the resource budget and communicates certain information to the superordinate who readjusts the budget, etc. Usually included in the subordinate's communication is information about the shadow prices associated with the constraints on organizational resources.

In general, budgeting methods are preferable to pricing methods because they are primal feasible. This means that at each iteration of the allocation reallocation process the current plan of how to use the resources and which activities to support meets all the constraints imposed on the superordinate and the subordinates. Therefore, if any procedure for finding the optimal solution via a budgeting is stopped before an optimal solution is identified, the current solution can be implemented. This is unlike pricing methods which are generally dual feasible, but not primal feasible except at optimality. This is especially important in modelling organizations which iterate only a few times before implementing a program. Resource budgeting methods can accomplish coordination under very general conditions, at least in theory. In addition, they are used extensively by many organizations. One can summarize this by the following proposition:

<u>Proposition 2</u>

In a cooperative organization resource budgeting methods are preferable to pricing methods for coordinating resource allocation decisions. However, as this chapter pointed out the procedures for describing the step by step coordination process to find an optimal solution are realistic from a behavorial veiwpoint only in certain cases.

The discussion of the coordinating procedures in this chapter also suggest that it is very unlikely that the superordinate will select the optimal subordinate budgets in one iteration. This suggests proposition 3.

Proposition 3

Top management (the superordinate) must receive information about the effects of its allocation of resources on the subordinate's performance if an optimal objective solution is to be found. In other words, if management allocates resources based on information about just one alternative, e.g., its allocation last period, an optimal solution is unlikely to be found.

CHAPTER V

INTRODUCTION TO NON-COOPERATIVE ORGANIZATIONS

In this chapter the concept of a non-cooperative system in hierarchical decentralized organizations is introduced. By studying and critically analyzing the behavioral assumptions embodied in coordination mechanisms for a cooperative system, an argument is set forth for mechanisms which do not embody the limitations inherent in cooperative coordination mechanisms. Chapters III and IV reviewed the state of the art for pricing and resource budgeting coordination mechanisms. In this chapter it is demonstrated that if either one of these mechanisms is used for coordination in a non-cooperative hierarchical decentralized organization, "poor" resource allocation decisions may result. Finally, the underlying logic of goal and constraint intervention coordination mechanisms for non-cooperative organizations is discussed, and a case is made for a negotiation approach which explicitly recognizes that resource allocation decisions are dependent upon the actions of both the superordinate and the subordinates.

Analysis of Cooperative Organizations

In a cooperative system there does exist conflict within the organization; however, this conflict is between subordinates for limited resources. Each subordinate recognizes that his decisions are only a subset of all decisions which are made in the organization. He believes that what is good for the organization as a whole is good for him. Hence, his behavior is directed at doing the best he can with whatever resources he is budgeted. He believes that the share of the organizational resources allotted to him is commensurate with his contribution to the overall objective function.

There is no conflict between objective functions in a cooperative system. The superordinate's utility is reflected in the subordinate's objective function, or he assumes that his utility is reflected in the subordinates' objective functions. The important observation is the lack of conflict between superordinate and subordinate's objective functions. Pricing and budgeting mechanisms are designed for resolving only the conflict among subordinates over resources.

In Chapter I the concepts of absolute and relative coordination were introduced. Absolute coordination implies a mechanism by which the superordinate's objective function is maximized, while relative coordination refers to a mechanism for ensuring that a resource allocation plan is found which is satisfactory to the superordinate. Both pricing and resource budgeting approaches aim at the same objective: to maximize the overall objective function. As such these techniques imply that the only satisfactory alternative is the optimization of the overall criterion function. Thus, pricing and resource budgeting methods have been designed to accomplish only absolute coordination. This assertion is not surprising since the algorithms for the case were generally designed as methods for decomposing a large mathematical programming problem into smaller parts. Mathematical problems are usually considered as maximization and minimization problems, and thus assume that the decision maker whose problem is represented by the mathematical program is an economic man, e. g., see March and Simon [80, p. 137].

In order to extend the pricing and resource budgeting methods to allow relative coordination, one could redefine the overall objective so that it represents a satisficing problem. A way of doing this is given in Chapter VI. Another way to handle relative coordination is to proceed as in Chapter III and IV in maximizing an objective and to terminate the iterative procedure whenever a satisfactory objective function level is attained. Such an approach necessitates that the algorithm for coordination be primal feasible. Thus, resource budgeting algorithms would work, but most pricing algorithms would not because they are not primal feasible. An exception, of course, would be the Dantzig-Wolfe algorithm because it can be a primal feasible procedure.

Since the motivation underlying most price directive and resource directive algorithms is largely a computational one, i. e., how can one solve a large mathematical problem efficiently, the behavioral implications are not necessarily appealing. In fact, one can interpret pricing and resource budgeting mechanisms as merely schemes by which the subordinate decision makers can be manipulated to attain the same decision at which the superordinate would have arrived, had he known all the constraints. Thus, a subordinate exists as a tool for carrying out the desires of the superordinate.[22] The subordinate in effect has no autonomy and no impact on the final allocation decision. His purpose is to process information and solve problems. Under these circumstances pricing and resource budgeting methods certainly do not enhance the motivational aspects of decentralized decision making. The superordinate's task is really to create an environment in which the subordinate believes he is participating in the decision making process. The conclusion of this discussion is that because of the manipulative intent pricing and resource budgeting mechanisms are not as appealing from a behavioral viewpoint as the literature might indicate.

Another pertinent observation regarding models of coordination for a cooperative system is that the structure of the organization has no effect on the final set of decisions. Thus, regardless of the organizational structure the final solution is the same. The subordinate decision making units can be combined with no effect on the solution. Again, this result is due to the fact that pricing and resource budgeting methods were designed for decomposing large mathematical problems. The following proposition is suggested.

<u>Proposition 4</u>

Suppose an organization is cooperative. If a pricing or resource budgeting coordination mechanism is used, the final resource allocation program selected for implementation is unaffected by the structure of the organization.

Empirical studies such as those discussed by Machlup [77] add

[22] This would agree with Taylor's concept of scientific management [80,p.13].

validity to the assertion that the actual decisions made in a hierarchical decentralized organization are not necessarily the decisions which would advance the objectives of the superordinate the most. However, according to the coordination models of Chapters III and IV the final decision should represent the objectives of the superordinate. One possible reason for this discrepancy between practice and theory is that most organizations are not cooperative, but instead can be described as non-cooperative systems.

Since the assumptions embodied in cooperative systems are not totally satisfactory and not totally realistic, an analysis of more general situations, viz., non-cooperative systems, seems justified. These more general cases should imply that the form and structure of the organization is a determining factor in the organization's decision behavior.

Before discussing coordination methods for non-cooperative systems, the next two sections indicate what the results can be if pricing or resource budgeting approaches are used in a non-cooperative system. The reader interested primarily in the mathematical aspects of the resource allocation coordination process in a decentralized organization may find the following two sections disconcerting. For example, it may be apparent that if some procedure for coordinating is used by the subordinate with respect to his utility function, but the subordinate uses a different utility function, then a suboptimal solution is reached. However, in the context of organizations this is not obvious. The language of mathematics brings to bear a certain clarity which can facilitate the statement of propositions which make contributions in the area of organizational structure and design.

Failure of Pricing Methods

To the author's knowledge no published work has appeared which uses a pricing like mechanism for coordination when the superordinate's objective function is different from that of the subordinates. This section indicates that except under very strong assumptions, the pricing approach will lead to a feasible solution, but the solution will not be optimal with respect to the superordinate's or the subordinate's objective function.

Consider a general model for two divisions. The superordinate perceives the resource allocation problem to be of the form:

$$\text{maximize} \quad g_1(\underline{x}_1) + g_2(\underline{x}_2) \tag{5-1}$$

$$\text{subject to:} \quad \underline{h}_1(\underline{x}_1) + \underline{h}_2(\underline{x}_2) \leq \underline{b} \tag{5-2}$$

$$\left.\begin{array}{ll} \underline{a}_1(\underline{x}_1) & \leq \underline{C}_1 \\ \qquad \underline{a}_2(\underline{x}_2) & \leq \underline{C}_2 \end{array}\right. \tag{5-3}$$

$$\underline{x}_1,\ \underline{x}_2 \geq \underline{0}$$

Suppose the objective function, (5-1), is composed of two strictly concave differentiable functions and is determined by the superordinate. The constraints concerning organizational resources are given by (5-2) where $\underline{h}_i(\underline{x}_i)$ is a vector of convex differentiable functions. The constraints in (5-3) are local restrictions concerning each subordinate's activities and are assumed to be convex differentiable functions.

Finally, it is assumed that the constraints (5-2) and (5-3) satisfy a constraint qualification. Part of these assumptions are made to ensure that coordinating prices exist. In the problem above the variable, $\underline{x}_K$, is a vector of decision variables; however, it may be that the superordinate is only concerned with some aggregate function of $\underline{x}_K$, e. g., the profit associated with $\underline{x}_K$. It is not assumed that the constraints expressed in (5-3) are known explicitly by the superordinate, but only that the superordinate realizes there are certain local restrictions for each subordinate.

On the other hand each subordinate perceives the overall problem differently. He perceives (5-2) and (5-3) just as the superordinate but he has his own objective function. The sum of the subordinates' objectives if:

$$\text{maximize} \quad f_1(\underline{x}_1) + f_2(\underline{x}_2) \tag{5-4}$$

of course, subordinate K knows his local constraints explicitly, i. e., $\underline{a}_K(x_K) \leq \underline{c}_K$.

One might inquire under what conditions would the superordinate's solution be the same as the subordinates' solution. Such a question can be investigated using the optimality conditions for mathematical programming. Since the constraint sets of the superordinate's and the sum of the superordinates' problems are the same, any feasible solution to one is also feasible to the other. Hence, one need only worry about optimality.

Remark 5.1

If $\underline{g}_K(\underline{x}_K) = \alpha\, f_K(\underline{x}_K)$ for all K where α is a positive scaler, then the optimal solutions to the superordinate's problem and the subordinates' problems are equivalent.

This result is obvious since the optimal solution to any mathematical programming problem is unchanged if the objective function is multiplied by a positive scaler. This result implies that if the superordinate consistently overestimates or underestimates the value of projects, then a pricing approach can bring about absolute coordination.

A prime difficulty in a pricing approach for a situation such as that in (5-1) to (5-4) is how can the superordinate determine when to stop communicating prices, $\underline{\lambda}$, to the subordinates, i. e., when has an optimal solution with respect to the superordinate's objective been found? The Lagrangian form of the superordinate's problem is:

$$\min_{\underline{\lambda}} \left\{ \max_{\underline{x}_1 \varepsilon X_1} [g_1(\underline{x}_1) - \underline{\lambda}'\underline{h}_1(\underline{x}_1)] + \max_{\underline{x}_2 \varepsilon X_2} [g_2(\underline{x}_2) - \underline{\lambda}'\underline{h}_2(\underline{x}_2)] + \underline{\lambda}'\underline{b} \right\} \qquad (5\text{-}5)$$

where $X_K = \{x_K \mid \underline{a}_K(\underline{x}_K) \leq \underline{C}_K, \underline{x}_K \geq \underline{0}\}$ for K=1,2. The inner maximization operations in (5-5) are supposedly performed by the subordinates. However, subordinate K solves the problem

$$\max_{\underline{x}_K \,\varepsilon\, X_K} \quad f_K(\underline{x}_K) - \underline{\lambda}' h_K(\underline{x}_K)$$

since he perceives the problem differently than the superordinate. Thus, if subordinate K arrives at solution $\underline{x}_K^o$ for a given $\underline{\lambda}^o$, the superorginate must ascertain whether this solution

$$\underset{\underline{x}_K \in X_K}{\text{maximizes}} \quad g_K(\underline{x}_K) - \underline{\lambda}^{o\prime}\underline{h}_K(\underline{x}_K) \tag{5-6}$$

To determine this the superordinate can use Kuhn Tucker conditions and compute

$$D(\underline{\lambda}^o) = \nabla g_K(\underline{x}_K^o) - \underline{\lambda}^{o\prime}\ \nabla\underline{h}_K(\underline{x}_K^o) - \underline{\mu}_K^{o\prime}\ \underline{a}_K(\underline{x}_K^o)$$

where $\underline{\mu}_K^{o\prime}$ is the vector of optimal dual variable values associated with subordinate K's local constraints, $\underline{a}_K(\underline{x}_K) \leq \underline{C}_K$. If $\underline{x}_K^{o\prime}D(\underline{\lambda}^o)=0$, then the superordinate can conclude that $\underline{x}_K^o$ does maximize (5-6). However, if $\underline{x}_K^{o\prime}D(\underline{\lambda}^o) \neq 0$, then $\underline{x}_K^o$ does not maximize (5-6). The important conclusion is that for the superordinate to determine whether a solution is optimal with respect to his objective function, he must have information about the gradients of the subordinates' constraints and the dual variable values for the subordinates. There is no way to attach any meaningful interpretation in terms of decentralized planning to such a requirement. One must conclude in agreement with Arrow that, "The top management can never, strictly speaking, know if the activity manager's objective function has been maximized." [9, p. 400].

Even more serious than the requirement that the superordinate have additional information about the subordinate's problems is that even if the superordinate has complete information about the subordinates' constraints, it may be impossible to achieve absolute coordination. While this may be intuitively obvious, a simple example problem is given to demonstrate this fact.

Example 5.1

Superordinate's objective function : $-(x_1-4)^2-(x_2-1)^2$

sum of subordinates' objective functions: $-(x_1-1)^2-(x_2-4)^2$

subject to:

$$x_1 + x_2 \leq 3$$

$$x_1 \leq 2$$

$$x_1, x_2 \leq 0$$

The solution $x_1^* = 2$, $x_2^* = 1$ is optimal with respect to the superordinate's objective function. Thus, the superordinate seeks a price, λ, so that subordinate 1 and 2 will find the superordinate's optimal solution. Subordinate 2's problem is:

$$\text{maximize} \quad -(x_2-4)^2 - \lambda x_2 \qquad (5\text{-}7)$$

$$x_2 \geq 0$$

If $x_2 = 1$ is to be optimal then (5-7) must have a stationary point at $x_2 = 1$. Therefore,

$$-2(x_2-4) - \lambda = 0$$

$$\lambda = -2(x_2-4) = -2(-3) = 6.$$

Thus, if $\lambda = 6$ subordinate 2 finds $x_2 = 1$. On the other hand, if

$\lambda = 6$ subordinate 1's problem is

$$\text{maximize} \quad -(x_1-1)^2-6x_1$$

$$\text{subject to} \quad x_1 \leq 2$$
$$x_1 \geq 0$$

The solution is $x_1 = 0$ which is not the superordinate's optimum. Thus, for this example it is not possible to find a price which will cause each subordinate to find the superordinate's optimum.

This discussion can be summarized in the following propositions.

Proposition 5

In a non-cooperative organization where pricing mechanisms are used, the superordinate may be unable to identify when an optimal solution has been reached. Except when a subordinate's utility function is a multiple of the superordinate's utility function, the superordinate requires additional information about the subordinate's constraint set in order to ascertain whether a solution is optimal.

Proposition 6

Even if the superordinate has complete information, it may be impossible to use pricing approaches for accomplishing absolute coordination in a non-cooperative organization.

It is interesting to note what would happen if a pricing algorithm is used in an organization where the superordinate's objective differs from the sum of the subordinates' objectives. Such a situation is of interest because it is quite conceivable that in a hierarchical decentralized organization the superordinate might believe that his subordinates have the same objective functions that he does, i. e., the superordinate might believe the organization is "cooperative".[23] For example, suppose a coordination mechanism is used which uses information flows in a similar way as the Dantzig-Wolfe algorithm. That is, the superordinate passes down prices to the subordinates. Given a pricing vector, subordinate K solves his problem using his utility function rather than the superordinate's utility function. The subordinate then communicates the solution he found or information about the effect of the solution on the superordinate's objective function. The superordinate assumes that subordinate K has found the communicated solution using the supremal's utility function. Therefore, he treats the communicated information in the same way that was done in the Dantzig-Wolfe algorithm.

Mathematically, it is clear that the solution found may not be optimal with respect to either the superordinate's utility function or the subordinate's utility functions. Suppose the overall problem is given by relationships (5-1) to (5-4) where all relationships are linear if the pricing algorithm discussed in the previous page is used, the overall problem which is really being solved is:

$$\text{maximize} \sum_{K=1}^{n} \underline{g}_K{}' \underline{x}_K^t \qquad (5\text{-}8)$$

[23] For example, a cooperative organization assumes that the superordinate and the subordinates have the same attitude toward risk. Studies where decision makers at different levels have different attitudes toward risk are currently being pursued. [109, 117]

$$\text{subject to:} \quad \sum_{K=1}^{n} H_K \underline{x}_K^t \leq \underline{b}$$

$$\underline{x}_K^t \in E_K$$

where E_K is the set of extreme points generated by subordinate K solving the problem:

$$\text{maximize} \quad (\underline{f}_K' - \underline{\lambda}' H_K)\underline{x}_K$$

$$\text{subject to:} \quad A_K \underline{x}_K \leq \underline{C}_K$$

$$\underline{x}_K \geq \underline{0}$$

Therefore, the claim that the use of a Dantzig-Wolfe like approach for coordinating in a non-cooperative organization does not work is not because of any mathematical or theoretical shortcomings, but simply because the "wrong" problem is being solved. This mathematical triviality is organizationally significant because often objective functions at different levels are different.

If a pricing scheme whereby new prices are generated by solving a master problem like (5-8) is used, it will converge, i. e., after a finite number of interations the superordinate will stop communicating prices and conclude that an optimal solution has been found. Unfortunately, the solution found is for the problem in (5-8).

Theorem 5.2

Suppose the resource allocation problem facing a non-cooperative organization is linear. If a pricing mechanism which utilizes information flow in a manner analogous to the Dantzig-Wolfe algorithm,[24] the iterative procedure will converge.

Proof: At iteration t of the procedure (analogous to Dantzig-Wolfe algorithm) subordinate K's problem is:

$$\text{maximize} \quad \underline{f}_K'\underline{x}_K - (\underline{\pi}_o^{t'} H_K)\, \underline{x}_K$$

$$\text{subject to:} \quad A_K\underline{x}_K \leq \underline{C}_K$$

$$\underline{x}_K \geq 0$$

The solution to the above problem yields extreme point $\underline{x}_K^t$. The superordinate's problem at step t+1 is:

$$\text{maximize} \quad \sum_{K=1}^{2} \sum_{K=1}^{t} [\underline{g}_K'\underline{x}_K^s]\, \lambda_K^s$$

$$\text{subject to} \quad \sum_{K=1}^{2} \sum_{s=1}^{t} [H_K\underline{x}_K^s]\lambda_K^t \leq \underline{b} \qquad (5\text{-}9)$$

$$\sum_{s=1}^{t} \lambda_K^s \qquad = 1 \text{ for } K=1,2 \qquad (5\text{-}10)$$

$$\lambda_K^s \geq 0$$

---

[24] The actual information is different in the proposed scheme. In the proposed method the subordinate generates extreme points using his own utility function while in Dantzig-Wolfe the subordinate generates extreme points using the superordinate's objective function.

The solution of the superordinate's problem yields the dual variables $\underline{\pi}_o^{t+1}$ and π_K^{t+1}, K=1,2, corresponding to constraints (5-9) and (5-10). To show that this process converges, it is only necessary to verify that at some iteration T the following holds:

$$\underline{\pi}_o^{T'} [H_K \underline{x}_K^T] + \pi_K^T - \underline{g}_K' \underline{x}_K^T \geq 0 \quad \text{for } K=1,2 \qquad (5\text{-}11)$$

If (5-11) does not hold for some K at iteration t, then the current value of the superordinate's objective function can be improved by bringing $\lambda_{K'}^t$ into the basis, this generating a new pricing vector $\underline{\pi}_o^{t+1}$. Since there exist only a finite number of extreme point solutions to each subordinate's problem,[25] for any pricing vector only a finite number of solutions can be generated. If the variable in the superordinate's problem corresponding to a subordinate's extreme point enters the basis, then it will never again be a candidate to enter the basis. Thus, in a finite number of steps, there will be no candidate subordinate solutions which can improve the superordinate's objective function, and therefore, (5-11) will be satisfied.

Theorem 5.2 demonstrates that a Dantzig-Wolfe like pricing coordination process will converge finitely when applied in a non-cooperative organization. However, the final solution may be non-optimal with respect to both the superordinate's objective function and the sum of the subordinates' objective functions. In fact it may converge to a solution which apriori of the solution process, the super-

[25] If a subordinate's constraint set is unbounded, then one must consider the extreme points and the extreme rays.

ordinate would feel is not satisfactory [60, p. 62]. Thus, in the event that the Dantzig-Wolfe approach is used where the superordinate's and the subordinates' objectives are not the same, it will often "appear" that the allocation process has converged; however, the final solution may be far from optimal.

If a price adjustment rule such as Uzawa's [114] is used (see Chapter III) for coordination in a non-cooperative organization, then the optimal solution for the sum of the subordinates' objective functions is found. Clearly, this solution is feasible for the superordinate, but it is seldom optimal. The reason that the superordinate's objective function has no influence is because the price adjustment rule administered by the superordinate depends only on the current solution generated by the subordinates, i. e., a new pricing vector, $\underline{\lambda}^{t+1}$ is determined by

$$\underline{\lambda}_j^{t+1} = \max[0, \lambda_j^{\,t} + \alpha \left(\sum_{i=1}^{n} h_i^{\,j}(\underline{x}_i^{\,t}) - b^j \right)]$$

where λ_j^{t+1} is the j^{th} component of $\underline{\lambda}^{t+1}$. Thus, if a price adjustment rule is used when the superordinate's objective function differs from the sum of the subordinates' objectives, the superordinate has no influence on the solution process or on the final solution.

Proposition 7 summarizes the conclusions regarding use of a pricing coordination mechanisms.

Proposition 7

Pricing approaches are not suitable mechanisms for coordination in a non-cooperative organization. Since in Chapters I and II an agrument was made that differing goals between decision units in an organization is a common occurrence, pricing approaches would seem to offer little help in achieving coordination in most organizations.

Failure of Resource Budgeting Mechanisms

As with pricing approaches, resource budgeting methods can accomplish absolute coordination only when the set of optimal solutions to the organizations problem using the superordinate's objective function is the same as the set of optimal solutions using the sum of the subordinates' objective functions. No longer are the results of Chapter IV relevant, i. e., the superordinate's problem cannot be stated as problem R'. Clearly, the superordinate is not interested in the function $v_i(\underline{\alpha}_i)$ because this function represents the change in subordinate i's objective function as the resources are varied. Instead, the superordinate is interested in $w_i(\underline{\alpha}_i)$ which is the part of his objective controlled by subordinate i as a function of the resources given to subordinate i. The difficulty is that $w_i(\underline{\alpha}_i)$ is not known explicitly, and there is no apparent way to build up on approximation to the superordinate's objective function. The shadow prices associated with the subordinates' problems are of no benefit since they represent the change in the subordinate's objective function.

As with pricing techniques it is interesting to understand what would happen if a resource budgeting algorithm is used to coordinate activities when the superordinate's objective differs from the sum of the subordinates' objectives. To facilitate this study the following resource budgeting scheme is utilized. The superordinate establishes resource budgets for each subordinate. In light of his budget, subordinate i solves the following problem:

<u>Problem $R_i(\alpha_i)$</u>

$$\text{maximize} \quad f_i(\underline{x}_i)$$

$$\text{subject to} \quad \underline{h}_i(\underline{x}_i) \leq \underline{\alpha}_i \qquad (5\text{-}12)$$

$$\underline{x}_i \quad \varepsilon \; X_i$$

He then communicates $\underline{\pi}_i$, the vector of dual variables associated with (5-12) and information about how the subordinate's current solution relates to the superordinate's objective. For example, if the superordinate is interested in maximizing $g_i(\underline{x}_i)$, then subordinate i might communicate $g_i(\bar{\underline{x}}_i)$ where $\bar{\underline{x}}_i$ is the solution found by solving problem $R_i(\alpha_i)$. The superordinate uses this information to adjust the subordinate's budgets, $\underline{\alpha}_i$. Since it has been shown in Chapter IV that a tangential approximation procedure requires only information about the subordinates' dual variables and information about the objective function value, this procedure is used. Using this scheme the following remarks can be made:

(1) It is no longer true that lower bounds exist on the optimal overall objective function value. Therefore, the information concerning how far from optimality a current solution is, has been lost.

(2) The sequence $\{\sum_{K=1}^{n} \sigma_i^K\}$ K=0,1,2,... v is still monotonically

decreasing because each time a subordinate's problem is solved a constraint is added to the superordinate's problem. Thus, as the procedure iterates the value of the superordinate's objective function, $\sum_{i=1}^{n} \sigma_i$, must decrease or stay the same.

(3) For a linear problem the above scheme will converge in the sense that at some point it is impossible to find a new set of $\underline{\alpha}_i$'s which is feasible and can improve the superordinate's objective function value. However, this solution may not be optimal with respect to either the superordinate or the subordinates. The reason for the apparent convergence is that a linear problem has only a finite number of extreme points or rays, and therefore only a finite number of constraints will be generated in the superordinate's problem.

(4) For a nonlinear problem the above scheme will not, in general, find the optimal solution.

These points should be mathematically obvious because a coordination procedure which was developed for a single problem is being used on two different problems. However, to further illustrate consider the following example:

Example 5.2

Superordinate's objective function:

$$\text{maximize } g_1(\underline{x}) + g_2(\underline{y})$$

$$\text{or maximize } 4x_1 + 3x_2 + y_1 + y_2$$

Superordinate's constraints: $\alpha_1 + \alpha_2 \leq 6$

$$\alpha_1, \alpha_2 \geq 0$$

Subordinate 1's objective: maximize $f_1 = x_1 + 2x_2$

Subordinate 1's constraints:

$$x_1 + x_2 \leq \alpha_1$$
$$5x_1 + 3x_2 \leq 15$$
$$3x_1 + 5x_2 \leq 15$$
$$x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

Subordinate 2's objective: maximize $f_2 = 4y_1 + 3y_2$

Subordinate 2's constraints:

$$y_1 + y_2 \leq \alpha_2$$
$$2y_1 + y_2 \leq 4$$
$$y_2 \leq 2$$
$$y_1, y_2 \geq 0$$

Using the superordinate's objective function the optimal solution is:

$x_1^* = 1.875$	$g_1^* = 13.125$
$x_2^* = 1.875$	$\alpha_1^* = 3.75$
$y_1^* = 1$	$g_2^* = 5$
$y_2^* = 2$	$\alpha_2^* = 2.25$

While in terms of the subordinates' objective function the optimal solution is:

$\bar{x}_1 = 1$	$f_1^* = 5$
$\bar{x}_2 = 2$	$\bar{\alpha}_1 = 3$
$\bar{y}_1 = 1$	$f_2^* = 10$
$\bar{y}_2 = 2$	$\bar{\alpha}_2 = 3$

Table 3 illustrates the results of using a tangential approximation approach. The scheme terminates with the solution: $x_1 = 1.67$, $x_2 = 2$,

Table 3. Summary of Example 5.2's Solution.

Iteration	σ_1	σ_2	α_1	α_2	f_1	f_2	g_1	g_2	π_1	π_2	Constraints Derived
1	-	-	0	0	0	0	0	0	2	4	$\sigma_1 \leq 2\alpha_1$ $\sigma_2 \leq 4\alpha_2$
2	0	24	0	6	0	10	0	5	2	0	$\sigma_1 \leq 2\alpha_1$ $\sigma_2 \leq 5$
3	9.5	5	4.75	1.25	5.67	5	12.67	1.25	0	4	$\sigma_1 \leq 12.67$ $\sigma_2 \leq -3.75 + 4\alpha_2$
4	7.625	5	3.8125	2.1875	5.67	8.315	12.67	3.562	0	2	$\sigma_1 \leq 12.67$ $\sigma_2 \leq -1.8124 + 2\alpha_2$
5	10.0625	.1250	5.0312	.9688	5.67	3.8752	12.67	.9688	0	4	$\sigma_1 \leq 12.67$ $\sigma_2 \leq -2.9064 + 4\alpha_2$
6	10.0625	.1250	5.0312	.9688	5.67	3.8752	12.67	.9688	0	4	

$y_1 = .9688$, $y_2 = 0$. The procedure is carried out in the following way. The superordinate initially chooses an $\underline{\alpha}_1$ and $\underline{\alpha}_2$ which satisfies his constraints. Using $\underline{\alpha}_K$ subordinate K solves his problem using his own utility function. He then communicates to the superordinate the solution found or the value (in terms of the superordinate's objective function) of the solution and the value of the dual variable associated with the resource constraints. This dual variable represents the change in the subordinate's objective function value per unit change in the available resource. For the example, the solution's value in terms of the objective functions of the superordinate and the subordinates is 9.5419 and 12.6355 respectively. Clearly, this solution is far from optimal for either the superordinate or the subordinates. In fact, if $\alpha_1 = 3$ and $\alpha_2 = 3$, the subordinates will find a solution which represents a value of 10 to the superordinate and a value of 15 to the subordinates which is better for both the superordinate and the subordinates than the solution found. In practice the superordinate would be likely to settle on $\alpha_1 = 3.8125$, $\alpha_2 = 2.1875$ (the solution at iteration 4) because it generates the greatest payoff in terms of the superordinate's objective function.

This example supports assertions (1), (2), and (3) made earlier. The mechanics of the example would be puzzling to the superordinate because the solution process terminated with a solution which yielded a value less than an earlier feasible solution. Such a happening might alert the superordinate to the fact that the subordinate is not optimizing with respect to the superordinate's objective function.

Figures 5 and 6 show the superordinate's and the subordinate's objective function as a function of the resource budget. Also, the approximation to the objective function as derived via the tangential approximation scheme is shown. Clearly, the approximating function represents neither the superordinate's nor the subordinate's response. In fact, the approximating function is not a convex combination of the superordinate's response and the subordinate's response. Examples of nonlinear problems seem to suffer the same difficulties. The conclusion can be stated in the following proposition.

Proposition 8

Resource budgeting approaches are not suitable mechanisms for coordination in a non-cooperative organization.

In a non-cooperative organization the iterative resource budgeting process maintains a solution which is feasible, but it may lead to a solution which is non-optimal with respect to the superordinate's objective. In fact, the process may find a solution which is dominated by another solution with respect to both the subordinates' and the superordinate's objective function. This suggests that a resource budgeting process may find "poor" solutions.

Basic Ideas of Goal and Constraint Intervention Methods

When the superordinate's and the subordinates' objective functions differ, a conflict often occurs because the subordinate wishes to select a decision from his constraint set which is not optimal for the superordinate. The superordinate wishes the subordinate

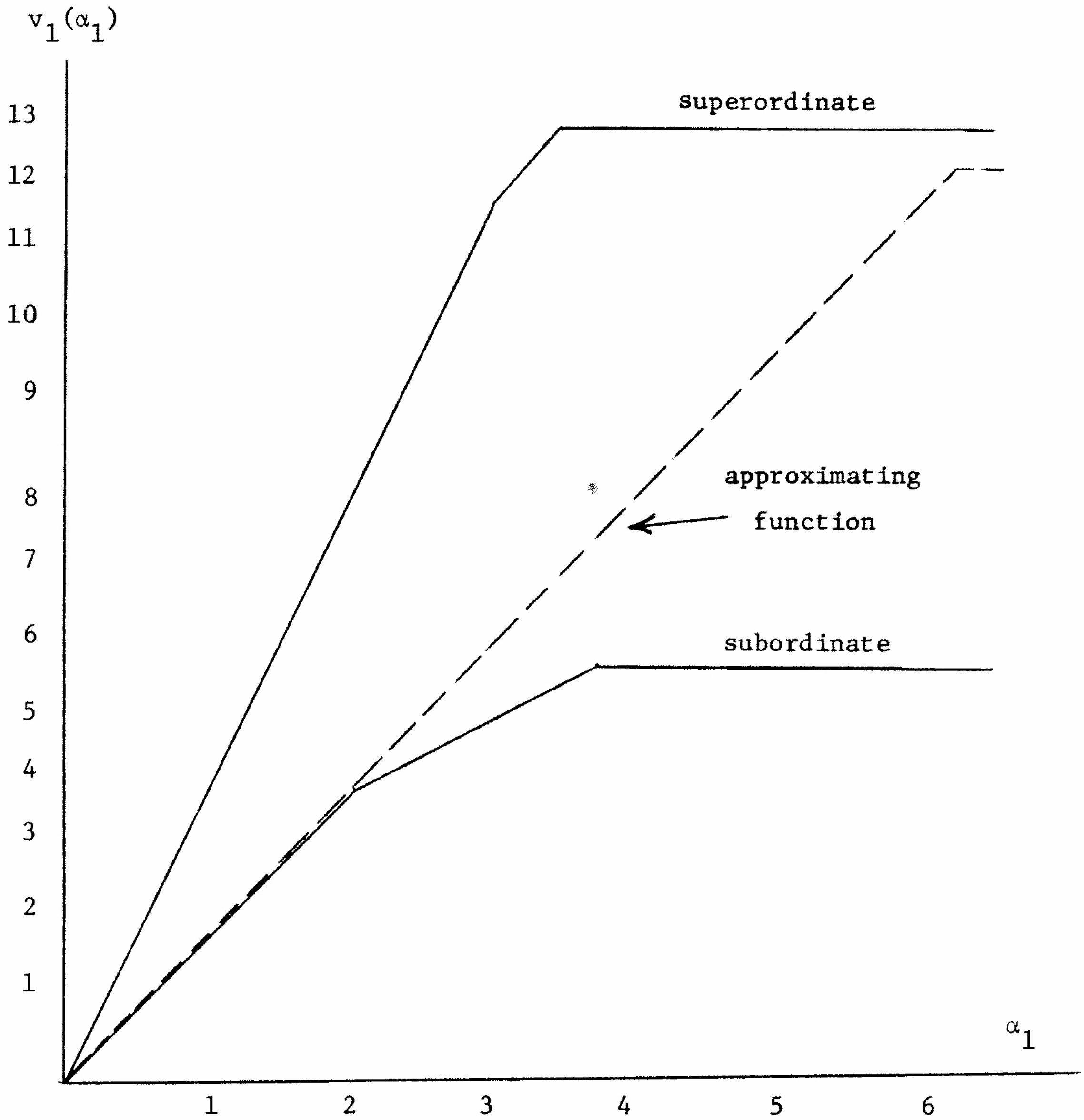

Figure 5. Subordinate 1's $v_1(\alpha_1)$

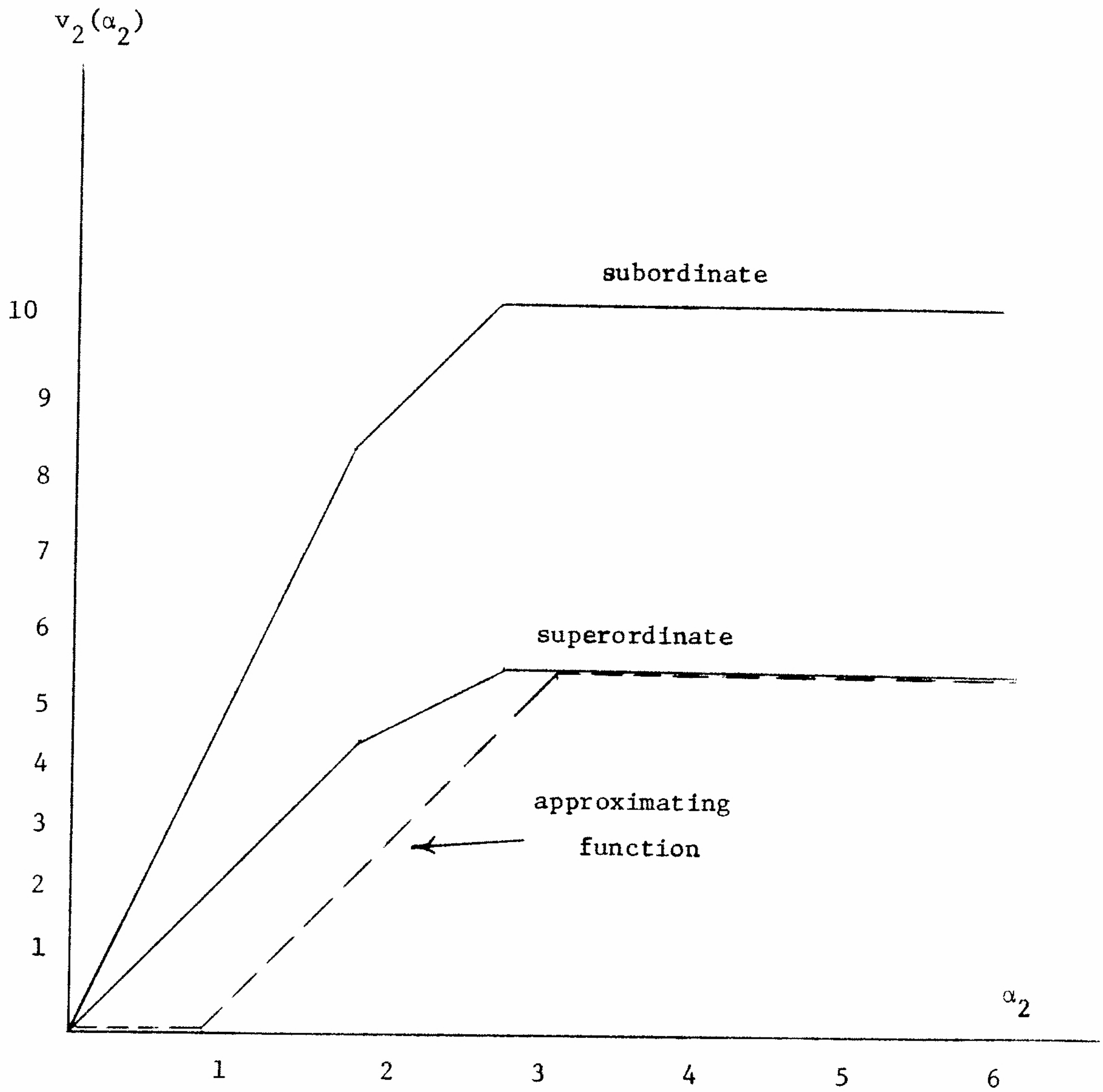

Figure 6. Subordinate 2's $v_2(\alpha_2)$

to select a decision which is located at some other point in the subordinate's constraint set. For example, in Figure 7 the optimal solution with respect to the superordinate's objective function is x* while the subordinate's optimum is y*. In general, the superordinate does not know explicitly the subordinate's constraint set.

Goal intervention methods allow the superordinate to alter the form of a subordinate's objective function. For example, in a non-cooperative organization pricing methods attempt to accomplish this by charging a price for an organizational resource, thus changing the subordinate's objective function. In general, the intent of goal intervention is to use meaningful incentives and other means to cause the subordinate to arrive at a solution (on his own) which is closer to the superordinate's optimum. Clearly, one means of accomplishing this is to "demand" that the subordinate use a specific objective function. There are other more subtle means such as tax incentives, profit sharing and salary compensation plans which still allow the subordinate some autonomy. For example, Kriebel and Lave [67] have investigated the effect of salary compensation plans and profit sharing when the superordinate seeks to maximize profit and the subordinate seeks to maximize utility which is a function of salary and output.

On the other hand, constraint intervention methods allow the subordinate to alter the subordinate's constraint set. For example, in a non-cooperative organization resource budgeting methods attempt to force the subordinate to select x* in Figure 7 by controlling the constraints on available resources. The effect of such action is to

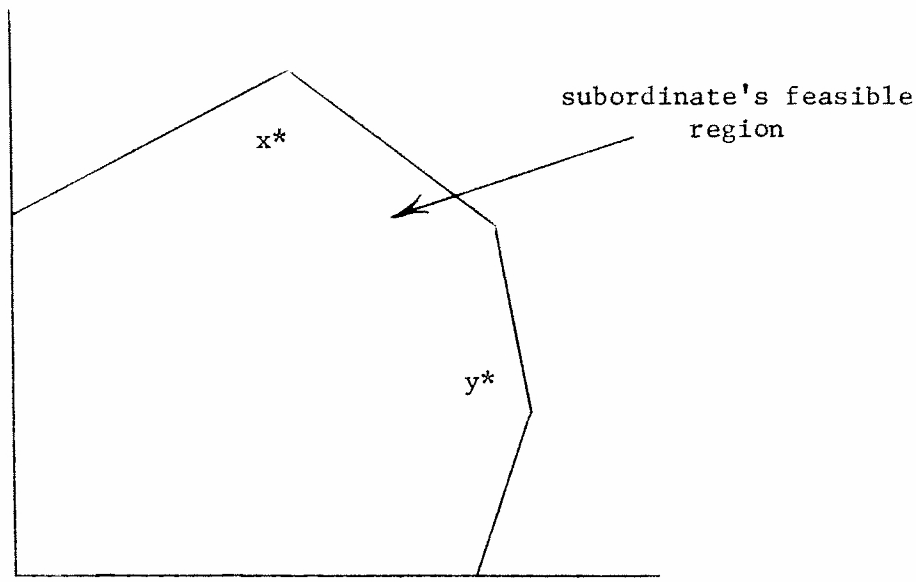

Figure 7. Subordinate's Set of Feasible Solutions.

alter the feasible region of a subordinate. However, as the previous section demonstrated, it is not always possible to ensure that this manipulation will cause the subordinate to select x*. Constraint intervention techniques allow the superordinate to impose meaningful constraints on the subordinate's constraint set which results in the subordinate finding x* or some solution which is satisfactory to the superordinate. Thus, constraint intervention methods provide the superordinate with more freedom in changing a subordinate's feasible set of decisions. Obviously, if the superordinate knew his optimal solution, x*, it would be possible to impose restrictions on the subordinate to force him to arrive at x*. As Simon says, "If you allow me to set the constraints, I don't care who determines the objectives," [103, p. 6]. This case would represent the extreme where the superordinate practically dictates the solution. More realistically, the superordinate does not know his optimum x*, and he wishes to use constraints as a means of ensuring the subordinate finds a solution which is "near" x*.

An example of a constraint intervention method which assumes particular objective function forms for the superordinate and the subordinates is the model presented by Cassidy, Kirby, and Raike [24]. They propose a model for determining how a central government can most efficiently allocate resources among other levels of government. The model assumes that once a superordinate allocates resources to the subordinates, then decisions about resource usage at the subordinate level cannot be controlled. The objective function of the subordinates is to maximize the value of undertaking projects while the superordinate tries to minimize the "relative regret" of the subordinates. In a sense the relative regret of a subordinate is a measure of how displeased the subordinate is with the funding he receives. The relative regret is measured as the difference between the value of a given budget and the value he would attain if given an unlimited budget. In this sense the minimization of relative regret does achieve the "best" distribution of resources. Dynamic programming is used to solve the problem [24, p. 467].

Summary

As a prelude to discussion of non-cooperative hierarchical decentralized organizations the assumptions inherent in a cooperative system were discussed. The important points were:

(1) The only conflict in the organization is a result of subordinates competing for limited resources.

(2) The structure of the organization has no effect on the final solution reached.

(3) The subordinates have no autonomy.

(4) The allocation decisions which are generated by a coordination procedure reflect only the objectives of the superordinate.

Each of these points limits the descriptive value of the models (e. g., [77, 123] present empirical evidence which indicates that point (4) above does not hold).

Next, it was shown that in a non-cooperative system where conflict also exists between the objectives of the superordinate and the subordinates that pricing and resource budgeting approaches fail to bring about coordination. In fact, if these approaches are used by a superordinate who assumes that the organization is cooperative, the iterative allocation process may appear to converge. However, the procedure may lead to a poor decision with respect to both the objectives of the superordinate and the subordinates. These arguments were made by reducing the superordinate's and the subordinates' problems to mathematical programming problems. From a mathematical standpoint the analysis is very simple and may appear trivial; however, from the viewpoint of someone interested in structuring information flow and coordinating allocation decisions in an organization the analysis and results are quite significant. Specifically, if an organization is non-cooperative there is little hope of finding "good" resource allocation plans through the use of pricing or budgeting techniques.

The basic intent of goal intervention and constraint intervention methods for non-cooperative systems was presented. Rather than attempt an exhaustive treatment of these coordination mechanisms, the next chapter will present and analyze a model for coordination in a non-cooperative organization. The model is referred to as a negotiation method because it results in a final decision which is influenced by the objectives of the superordinate and the subordinates.

CHAPTER VI

NEGOTIATION MODELS FOR COORDINATION IN NON-COOPERATIVE ORGANIZATIONS

The term negotiation model is borrowed from game theory, (see Luce and Raiffa [76, p. 118]) and is used because the processes for arriving at a resource allocation program are affected by both the superordinate and the subordinates. As March and Simon [80, p. 156] noted when the decision units participating in the organization's decision making process have different goals, then the decisions are reached by a predominately bargaining process. Thus, the superordinate and the subordinates influence the behavior of each other, and the final solution exhibits this. This is in contrast to pricing and resource budgeting approaches where both the superordinate and subordinates participate, but each works toward its own objective ignoring the other participants in the process.

In this chapter a "goal decomposition" approach suggested by Ruefli is reviewed. Although Ruefli's model significantly extends the state of art in coordination for non-cooperative organizations, there are some weaknesses in the interpretations associated with the iterative process. Using a procedure which is related to work by Kelley [62], Benders [21], Dantzig [37], and Zangwill [125], a goal partitioning procedure is presented. The theoretical and behavioral properties of the goal partitioning procedure model are discussed and shown to provide a description of the resource allocation process in hierarchical decentralized organizations.

A Goal Decomposition Method

Ruefli in his doctoral dissertation [90] developed a powerful analytical approach for representing the resource allocation decision process in a three level organization. The decisions at the levels are respectively: generation of resource budgets and performance targets, evaluation and selection of alternatives, and the generation of alternatives. Figure 8 depicts Ruefli's three level process. In keeping with the previous strategy of investigating two level structures, the following discussion is limited to the decisions of generation of resource budgets and performance targets and the evaluation and selection of alternatives. Ruefli's model is significant because it is the first approach in which the nature of the resource allocation decision is dependent upon the structure of the organization.

Ruefli's approach seems to incorporate many of the significant characteristics of the resource allocation process in the "real world". Utilizing Ruefli's model results in the allocation problem being expressed as an optimization problem. Ruefli proposes the use of generalized linear programming to solve this problem. [91, p. 509]

In studying Ruefli's model and solution procedures, this author discovered a serious limitation (from a behavioral viewpoint) associated with the use of generalized linear programming for solution of the allocation problem posed by Ruefli. Although generalized linear programming can be used to solve the overall problem posed by Ruefli, the interpretation of the procedure does not allow the subordinates to

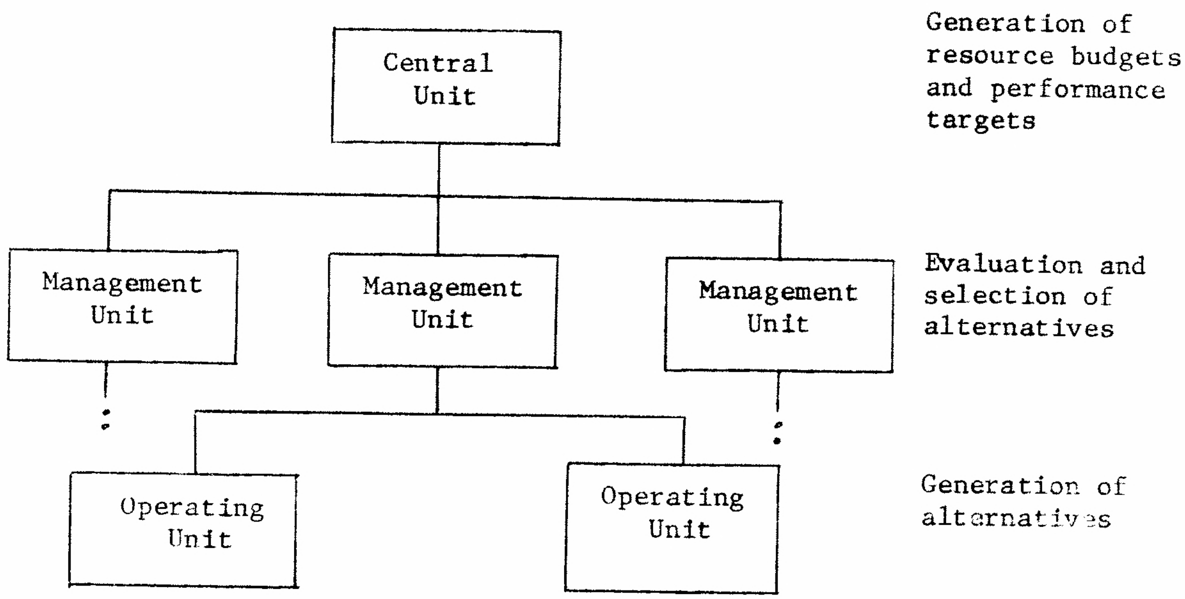

Figure 8. Ruefli's Three Level Organization.

function independently of each other, i. e., subordinate i cannot solve his allocation program without considering the resource allocation problems facing the other subordinates. This difficulty can be overcome by using a partitioning method rather than generalized linear programming. In the following paragraphs Ruefli's generalized goal decomposition model is described, and then an example is used to demonstrate the undesirability of using generalized linear programming.

In his development [90, 91] Ruefli first introduces the subordinate's problems, and then the superordinates problem is discussed. However, it is possible to state the overall problem of the organization first, and then derive the superordinate's and subordinates' problems. The overall problem can be stated as:

$$\text{minimize} \quad \sum_{K=1}^{n} [\underline{w}_K^{+\prime} \underline{d}_K^{+} + \underline{w}_K^{-\prime} \underline{d}_K^{-}]$$

$$\text{subject to:} \quad \sum_{K=1}^{n} P_K \underline{\alpha}_K \leq \underline{G}_o$$

$$\sum_{j=1}^{m_K} \underline{h}_{jK} x_{jK} - \underline{d}_K^{+} + \underline{d}_K^{-} - \underline{\alpha}_K = \underline{0} \quad \text{for } k=1,\ldots,n \qquad (6\text{-}1)$$

$$x_{jK} \leq 1 \quad \text{for } j=1,\ldots,m_K, \; K=1,\ldots,n$$

$$x_{jK} \geq 0, \; \underline{d}_K^{+}, \; \underline{d}_K^{-} \geq \underline{0}$$

where $\underline{\alpha}_K = P_K \times 1$ vector of goals for subordinate K,

x_{jK} = activity level of project jK,

$\underline{h}_{jK} = P_K \times 1$ vector of project attributes,

$\underline{d}_K^+, \underline{d}_K^- = P_K \times 1$ vectors of positive and negative deviations from the goals expressed in $\underline{\alpha}_K$, and

$\underline{w}_K^+, \underline{w}_K^- = P_K \times 1$ vectors of weights associated with the positive and negative deviations.

$\underline{G}_o = m_o \times 1$ vector of fixed goals

$P_K = m_o \times P_K$ matrix which relates subordinate K's goal vectors to the fixed goals, G_o.

The above overall problem can be decomposed in a straightforward manner. Notice that the constraints represented in (6-1) could be written equivalently as:

$$\sum_{j=1}^{m_K} \underline{h}_{jK} x_{jK} - \underline{d}_K^+ + \underline{d}_K^- - \underline{\alpha}_K z = \underline{0} \qquad (6\text{-}2)$$

$$z = 1 \qquad (6\text{-}3)$$

Now, the overall problem may be separated into two parts:

subordinate's portion

$$\text{minimize} \quad \sum_{K=1}^{n} [\underline{w}_K^+ \underline{d}_K^+ + \underline{w}_K^{-\prime} \underline{d}_K^-] \qquad (6\text{-}4)$$

$$\text{subject to:} \quad \left.\begin{aligned} \sum_{j=1}^{m_K} \underline{h}_{jK} x_{jK} - \underline{d}_K^+ + \underline{d}_K^- - \underline{\alpha}_K z &= \underline{0} \\ z &= 1 \end{aligned}\right\} k=1,\ldots,n$$

$$x_{jK} \leq 1$$

$$x_{jK} \geq 0, \underline{d}_K^+, \underline{d}_K^- \geq \underline{0}$$

The other portion, the superordinate's part, is to select vectors, $\underline{\alpha}_K$, such that $\sum_{K=1}^{n} P_K \underline{\alpha}_K \leq \underline{G}_o$. The motivation for the superordinate's choice can be shown by letting the dual variables associated with (6-2) and (6-3) be $\underline{\pi}_K$ and γ respectively. The usual simplex criterion to determine if a variable should enter the basis is: enter variable j if $z_j - c_j > 0$. In terms of the above problem this condition corresponds to $- \underline{\pi}_K'\underline{\alpha}_K + \gamma > 0$. Therefore, if one seeks to maximize $- \underline{\pi}_K' \underline{\alpha}_K + \gamma$ or equivalently minimize $\underline{\pi}_K' \underline{\alpha}_K$,[28] new values for $\underline{\alpha}_K$ can be generated. However, only certain $\underline{\alpha}_K$ values are permissible because of the constraints, $\sum_{K=1}^{n} P_K \underline{\alpha}_K \leq \underline{G}_o$. Thus the superordinate can generate new $\underline{\alpha}_K$'s by solving the problem

$$\text{minimize} \quad \sum_{K=1}^{n} \underline{\pi}_K'\underline{\alpha}_K$$

$$\text{subject to:} \quad \sum_{K=1}^{n} P_K \underline{\alpha}_K \leq \underline{G}_o.$$

$\underline{\pi}_K$ represents a $P_K \times 1$ vector of prices whose j^{th} element corresponds to the change in subordinate K's objective function per unit change in the j^{th} goal. Since the $\underline{\pi}_K$ vector corresponds to the vector of equality constraints, (6-2), it can assume positive or negative values. The interpretation associated with the kinds of information flows for the iterative solution procedure is similar to that of resource budgeting approaches.

[28] Once γ is determined, it can be treated as a constant.

A verbal description for Ruefli's model in the context of a decentralized organization is now given. The subordinates seek to determine what project levels should be undertaken in order to minimize the weighted deviation from the goals which have been set for them by the superordinate. Since the superordinate specifies the $\underline{\alpha}_K$'s in the subordinate's portion of the overall problem, he has the ability to alter the subordinates' set of feasible solutions. The weights, $\underline{w}_K^+$ and $\underline{w}_K^-$, are assumed to be fixed throughout the solution process; however, Ruefli is not clear on who (superordinate, subordinate, or both) sets the weights.

If the j^{th} project is not undertaken by subordinate K, then $x_{jK} = 0$. On the other hand, $x_{jK} = 1$ implies that subordinate K fully implements project j. The formulation allows for fractional x_{jK} values which corresponds to undertaking only portions of a project. If x_{jK} were required to be either zero or one, there is no satisfactory interpretation of the dual variables (see Balas [16], Gomory [52], and Alcaly [5]),and it will be shown that the solution algorithm depends on the usual dual variable interpretation. However, in many cases fractional x_{jK} values can have meaning, e. g., $x_{jK} = 1/2$ could mean that project jK is funded at one-half its maximum level. (See [4, 14] for illustrations of how such schemes can be incorporated) The fixed vector $\underline{h}_{jK}$ relates the characteristics of project jK to the goals $\underline{\alpha}_K$. Generally, the goal vector, $\underline{\alpha}_K$, is composed of resources and requirements. For example, $\underline{\alpha}_K$ could equal $(\alpha_{1K}, \alpha_{2K}, \alpha_{3K}, \alpha_{4K})'$ where α_{1K} = budget in dollars, α_{2K} = budgeted manpower, α_{3K} = profit, and α_{4K} = share of the market.

The superordinate's task is to select the vectors $\underline{\alpha}_K$, K = 1,...,n from some convex polyhedral set. Thus, the superordinate seeks to choose $\underline{\alpha}_K$, K = 1,...,n so that

$$\sum_{K=1}^{n} P_K \underline{\alpha}_K \leq \underline{G}_o$$

where the values of the components of $\underline{G}_o$ are often fixed by exogeneous factors such as government regulations, stockholders, union contracts, etc. There is a close relationship between this model and those models discussed in conjunction with resource budgeting mechanisms. Resource budgeting methods are concerned only with partitioning the resources among the subordinates in such a way so that supply is not exceeded, and the overall objective function is maximized. Ruefli's model can accomplish this, but, in addition, it is concerned with partitioning performance targets among the subordinates. Thus, a main extension made by Ruefli is to actually make explicit the consideration of both performance targets and resource budgets. As Malinvaud points out the main distinction between most models and reality is that models usually do not allow the planning bureau (superordinate) to fix production targets [78, p. 205].

In Ruefli's model the superordinate sets target levels or goals for each subordinate. These goals consist of resource budgets and performance requirements. In the mathematical formulation the target levels for a subordinate appear as the right hand side of constraint

(6-1). Therefore, the act of fixing the target level influences the set of solutions which the subordinate can undertake, assuming he does not deviate from the target levels. In effect, the superordinate is changing the subordinate's constraint set. Therefore, Ruefli's model is related to coordination through constraint intervention.

Given an overall organizational resource allocation problem Ruefli proposes that an iterative information exchange take place between the superordinate and the subordinates. During the exchange the superordinate selects goals, $\underline{\alpha}_K$, by solving his problem and communicates them to the subordinates. He advocates that each subordinate then solve the problem:

$$\begin{aligned}
\text{minimize} \quad & \underline{w}_K^{+'} \underline{d}_K^{+} + \underline{w}_K^{-'} \underline{d}_K^{-} \\
\text{subject to:} \quad & \sum_{j=1}^{m_K} \underline{h}_{jK} x_{jK} - \underline{d}_K^{+} + \underline{d}_K^{-} - \underline{\alpha}_K z = \underline{0} \qquad (6\text{-}2) \\
& x_{jK} \leq 1 \\
& z = 1 \\
& z,\ x_{jK} \geq 0,\ \underline{d}_K^{+},\ \underline{d}_K^{-} \geq \underline{0}
\end{aligned}$$

Upon solution of this problem subordinate K communicates the dual multipliers associated with (6-2) to the superordinate, and the process continues. The process described above may not converge to the correct solution and is not the correct way to apply generalized linear programming to the overall problem.

The correct interpretation is for the superordinate to select goals by solving his problem and then communicate them to the subordinates. The subordinates then, as a group, must solve their portion of the overall problem given by (6-4). In addition the subordinates must provide for taking a convex combination of the goals assigned iteratively by the superordinate. This is done by associating z^t with $\underline{\alpha}_K{}^t$ and replacing $z = 1$ by $\sum_t z^t = 1$. Thus, the subordinates' problem at iteration r of the iterative solution procedure is:

$$\text{minimize} \quad \sum_{K=1}^{n} [\underline{w}_K^{+'} \underline{d}_K^{+} + \underline{w}_K^{-'} \underline{d}_K^{-}]$$

$$\text{subject to:} \quad \sum_{j=1}^{m_K} \underline{h}_{jK}' x_{jK} - \underline{d}_K^{+} + \underline{d}_K^{-} - \sum_{t=1}^{r} \underline{\alpha}_K{}^t z^t = 0 \quad K=1,\dots,n$$

$$\sum_{t=1}^{r} z^t = 1$$

$$x_{jK} \le 1 \quad j=1,\dots,m_K \quad K=1,\dots,n$$

$$x_{jK} \ge 0, \ \underline{d}_K^{+}; \ \underline{d}_K^{-} \ge 0$$

This discussion illustrates that the interpretation associated with the solution of the problem represented by Ruefli's model does not allow the subordinates to operate independently, and the actual choice of the final goals is no longer decided by the superordinate. The behavorial implication is just the opposite of the Dantzig-Wolfe algorithm. Here the authority to make the final decision concerning the goal levels assigned to each subordinate is made by all the subordinates acting as a coalition. The superordinate communicates a new goal vector, $\underline{\alpha}_K$, to each subordinate at stage t of an iterative process. Each subordinate then considers this new goal vector along with all previous ones in determining what vector of goals will actually be assigned at iteration t. Thus, the superordinate participates in the decision process, but the final decision and the decision of which goals will be assigned to each subordinate is made by the subordinates acting as a group. In light of this interpretation, Ruefli's procedure is unappealing because it destroys the subordinates' independence. Fortunately, these problems can be overcome by another solution scheme which maintains the independence of the subordinates.

The solution procedure is really nothing more than an application of the Dantzig-Wolfe decomposition principle to the overall problem. However, the interpretation of superordinate's and subordinates' problem is simply reversed, i. e., the master problem in Dantzig-Wolfe is the subordinates' problem and the subproblem in Dantzig-Wolfe is the superordinate's problem. This reverse Dantzig-Wolfe process eliminates the behavioral drawback of the superordinate having to take convex combinations to arrive at a final solution. However, if an overall optimum is desired, it is necessary for the subordinates as a group to choose the convex combination. Ruefli never points out the serious drawback associated with this although he does indicate that the optimal goal vector, $\underline{\alpha}_K{}^*$, may lie in the interior of the superordinate's constraint set [91, p. 510]. Thus it is possible that

$$\underline{\alpha}_K^* = \sum_{t=1}^{T} \lambda_t \underline{\alpha}_K^t$$

where $\sum_{t=1}^{T} \lambda_t = 1$, $\lambda_t \geq 0$, and $\underline{\alpha}_K^t$ is the goal vector actually found by solving the superordinate's problem at iteration t, i. e., $\underline{\alpha}_K^t$ $(K=1,\ldots,n)$ is an extreme point of the set given by

$$\sum_{K=1}^{n} P_K \underline{\alpha}_K \leq \underline{G}_o.$$

Despite these limitations Ruefli's model is a contribution in the area of analytical techniques for modelling the decision process in a hierarchical decentralized organization. The reasons for this statement include:

(1) The model is among the first to combine concepts of organizational behavior and economic theory. For example, the idea of goal seeking behavior as displayed by the superordinate and the subordinates corresponds fairly well to the behavioral theory discussed by March and Simon [80].

(2) The solution methodology is based on the theory of linear programming for which simple and efficient solution procedures exist.

(3) The model was developed for a three level organization, but it can easily be extended to consider an n level organization.

(4) The model allows one to consider both technological and behavorial externalities. It has been noted [90, p. 56] that most pricing coordination mechanisms cannot handle externalities because they result in multiple prices for activities. Both behavioral and tech-

nological externalities imply some relationship between subordinate decision making units. An example of a technological externality might be if subordinate j implements a particular project, then subordinate K must implement a specific project. Ruefli would handle this difficulty by having the decision of whether to implement the project made by a unit which is superordinate to both subordinates. Thus, a technological externality tends to reduce the subordinates' decision making authority, and hence increase the amount of centralization of decision making. The concept of submitting the implementation decision to a higher authority is well known in management theory [105, p. 142].

Behavorial externalities arise when the goal seeking behavior of a subordinate is affected by the goal seeking behavior of another subordinate. For example, subordinate j might have as an objective to make a profit greater than subordinate K. Such dependencies create problems in proving that the goal decomposition procedure converges, e. g., see Ruefli [94] for a complete discussion.

(5) The solution for the resource allocation decision problem depends on the structure of the organization. This can be attributed to the use of a goal programming formulation. Ruefli illustrates this by considering allocation plans selected under two different structures: a PPBS structure and a bureacratic structure [93]. The allocation plan selected under each structure differed significantly. This supports the hypothesis that the formal structure of the information system and its relationship to the organizational structure is a

a determining factor for the behavior of the organization.

As further evidence Ruefli shows that the overall objective function and constraints will be different for a centralized and a decentralized structure [91, p. 514]. Thus, the resource allocation decision may differ under each structure.

To summarize, Ruefli has proposed an interesting model which would seem to represent some important aspects of the resource allocation process in hierarchical decentralized organizations; however, there are several difficulties which limit the effectiveness of his approach. In the next section these difficulties will be overcome.

Extensions to Ruefli's Model: A Goal Partitioning Procedure

To overcome the difficulties associated with the interpretation of the solution procedure for Ruefli's problem, a different approach is now given. This extension has its roots in the works of Kelley [62], Benders [21], Dantzig [36], Zangwill [125], and Geoffrion [46]. Consider the following as the problem facing the organization:

Organization's Problem

$$\text{minimize} \quad z = \sum_{K=1}^{n} [\underline{w}_K^{+\prime}\underline{d}_K^{+} + \underline{w}_K^{-\prime}\underline{d}_K^{-} + \underline{u}_K^{+\prime}\underline{e}_K^{+} + \underline{u}_K^{+\prime}\underline{e}_K^{-}]$$

$$\text{subject to:} \quad \left.\begin{array}{lll} H_K\underline{x}_K - \underline{d}_K^{+} + \underline{d}_K^{-} & - \underline{\alpha}_K & = \underline{0} \\ G_K\underline{x}_K & - \underline{e}_K^{+} + \underline{e}_K^{-} & = \underline{\beta}_K \\ A_K\underline{x}_K & & \leq \underline{C}_K \end{array}\right\} K=1,\ldots,n$$

$$\sum_{K=1}^{n} P_K \underline{\alpha}_K \leq \underline{q}$$

$$\underline{x}_K \geq \underline{0},\ \underline{d}_K^{+},\ \underline{d}_K^{-},\ \underline{e}_K^{+},\ \underline{e}_K^{-} \geq 0.$$

It is shown in the next paragraphs that the overall problem above can be decomposed into n subordinate problems and one superordinate problem. Subordinate K's decision problem is expressed as:

<u>Problem S_K</u>

$$\text{minimize} \quad z_K(\alpha_K) = \underline{w}_K^{+'}\underline{d}_K^{+} + \underline{w}_K^{-'}\underline{d}_K^{-} + \underline{u}_K^{+'}\underline{e}_K^{+} + \underline{u}_K^{-'}\underline{e}_K^{-}$$

$$\begin{aligned}
\text{subject to:}\quad & H_K\underline{x}_K - \underline{d}_K^{+} + \underline{d}_K^{-} && = \underline{\alpha}_K && (6\text{-}5)\\
& G_K\underline{x}_K - \underline{e}_K^{+} + \underline{e}_K^{-} && = \underline{\beta}_K && (6\text{-}6)\\
& A_K\underline{x}_K && \leq C_K &&
\end{aligned}$$

$$\underline{x}_K \geq \underline{0},\ \underline{d}_K^{+},\ \underline{d}_K^{-} \geq \underline{0},\ \underline{e}_K^{+},\ \underline{e}_K^{-} \geq \underline{0}$$

This problem statement differs from Ruefli's in the following ways:

(1) The subordinate has two types of goals, $\underline{\alpha}_K$ and $\underline{\beta}_K$. $\underline{\alpha}_K$ represents those resource budgets and performance targets which are set by the superordinate. $\underline{\beta}_K$ represents those goals which the subordinate has set for himself. The $\underline{\beta}_K$ goals remain fixed throughout the planning procedure, but the $\underline{\alpha}_K$ goals may be iteratively adjusted by the superordinate.

(2) H_K is a p_K x m_K matrix which relates the activities of

subordinate K to the goals included in $\underline{\alpha}_K$, and G_K is a $r_K \times m_K$ matrix relating the activities to the β_K goals.

(3) The constraints, $A_K\underline{x}_K \leq \underline{C}_K$, represent technological and other restrictions on subordinate K that must be satisfied. These constraints correspond to the "hard" constraints as discussed by Chamberlain [25] and Krouse [70]. In contrast to these constraints the restrictions in (6-5) and (6-6) are policy targets which may be internally adaptable. These changes are meant to improve the model's realism. They explicitly allow for the subordinate to have his own set of goals in addition to those set by the superordinate. In addition, the concept of two kinds of constraints: technical and adaptable, is included [35].

There are several interpretations that one may associate with the objective function for the subordinate. These interpretations are discussed later; however, for the present it is assumed that the value of a subordinate's objective function represents his discrepancy dissatisfaction. In this light the weights, $\underline{w}_K^+$, $\underline{w}_K^-$, $\underline{u}_K^+$, $\underline{u}_K^-$, represent the means for transforming the deviations which could be measured in dollars or man-hours, etc., into units of dissatisfaction. Thus, the weights represent the subordinate's priorities for the targets. This technique also handles the problem of multiple objectives.

The development for the goal partitioning procedure is now given. The dual of problem S_K is

<u>Problem D_K</u>

$$\text{maximize } v_K(\alpha_K) = \underline{\alpha}_K{}' \, \underline{\pi}_K + \underline{\beta}_K{}'\underline{\lambda}_K - \underline{C}_K{}'\underline{\mu}_K$$

$$\text{subject to:} \quad H_K'\underline{\pi}_K + G_K'\underline{\lambda}_K - A_K'\underline{\mu}_K \le \underline{0}$$

$$\begin{aligned} -\underline{\pi}_K & \quad \le \underline{w}_K^+ \\ \underline{\pi}_K & \quad \le \underline{w}_K^- \\ -\underline{\lambda}_K & \quad \le \underline{u}_K^+ \\ \underline{\lambda}_K & \quad \le \underline{u}_K^- \\ \underline{\mu}_K & \ge \underline{0} \end{aligned}$$

The superordinate's task is to divide up the resource and performance requirements among the subordinates in order to meet the overall restrictions imposed upon the organization, i. e., he must choose $\underline{\alpha}_K$, K=1,...,n so that

$$\sum_{K=1}^{n} P_K\underline{\alpha}_K \le \underline{q} . \qquad (6\text{-}7)$$

These overall restrictions can represent several different kinds of considerations. For example, limited resource supplies, desired profit levels, and requirements concerning the relationships between subordinates' resource budgets or performance levels, can all be handled via (6-7).

Since problem D_K is a linear programming problem, some optimum solution must occur at an extreme point. Notice that the extreme points do not depend on $\underline{\alpha}_K$. Thus, for any choice of $\underline{\alpha}_K$, the set of extreme points for the dual problem D_K does not change. Let $(\underline{\pi}_K^{S_K}, \underline{\lambda}_K^{S_K}, \underline{\mu}_K^{S_K})$ represent extreme points $S_K = 1,\ldots,E_K$ for problem D_K.

One might worry about whether problem D_K possesses a feasible bounded optimal solution. The following remarks address these questions.

Remark 6.1

If there exists an $\underline{x}_K$ such that $A_K\underline{x}_K \leq \underline{C}_K$, then problem D_K has a bounded optimum for any choice of $\underline{\alpha}_K < \infty$.

Proof: By hypothesis problem S_K possesses a feasible solution and thus according to linear programming duality theory, problem D_K cannot have an unbounded optimum.

Remark 6.2

If the organization's problem possesses a feasible bounded optimum solution, then problem D_K has a feasible solution for any choice of $\underline{\alpha}_K < \infty$ such that $\sum_{K=1}^{n} P_K\underline{\alpha}_K \leq \underline{q}$.

Proof: Infeasibility in problem D_K implies infeasibility or unboundedness in problem S_K. By hypothesis problem S_K cannot be infeasible because there is some x_K such that $A_K\underline{x}_K \leq C_K$. Problem S_K cannot have an unbounded optimum because this would imply that the organization's problem is unbounded.

From duality theory for a given $\underline{\alpha}_K$ any feasible solution, $\underline{x}_K$, $\underline{d}_K^+$, $\underline{e}_K^+$, $\underline{d}_K^-$, $\underline{e}_K^-$ to problem S_K has the property that $\underline{w}_K^{+\prime}\underline{d}_K^+ + \underline{w}_K^{-\prime}\underline{d}_K^- + \underline{u}_K^{+\prime}\underline{e}_K^+ + \underline{u}_K^{-\prime}\underline{e}_K^- \geq \underline{\alpha}_K'\underline{\pi}_K^{S_K} + \underline{\beta}_K'\underline{\lambda}_K^{S_K} - \underline{C}_K'\underline{\mu}_K^{S_K}$ for $S_K = 1,\ldots,E_K$ where $\sum_{K=1}^{n} P_K\underline{\alpha}_K \leq \underline{q}$. Also, given $\underline{\alpha}_K < \infty$, the optimal solutions to problems S_K and D_K have the property that $z_K^*(\alpha_K)=v_K^*(\alpha_K)$. Therefore, for a given $\hat{\underline{\alpha}}_K$

$$\sigma_K^* = z_K(\hat{\alpha}_K) \geq \hat{\underline{\alpha}}_K'\underline{\pi}_K^{S_K} + \underline{\beta}_K'\underline{\lambda}_K^{S_K} - \underline{C}_K'\underline{\mu}_K^{S_K} .$$

The organization's problem can then be written as

Problem 0

$$\text{minimize} \qquad \sigma = \sum_{K=1}^{n} \sigma_K$$

$$\text{subject to:} \quad \sigma_K \geq \underline{\alpha}_K{}'\underline{\pi}_K^{S_K} + \underline{\beta}_K{}'\,\underline{\lambda}_K^{S_K} - \underline{C}_K{}'\underline{\mu}_K^{S_K} \quad \text{for } S_K = 1,\ldots,E_K$$

$$K=1,\ldots,n$$

$$\sum_{K=1}^{n} P_K\,\underline{\alpha}_K \leq q$$

Theorem 6.3 (Benders, [21])

Problem 0 and the organization's problem are equivalent. The proof is given by Benders [21, p. 240] and is an immediate consequence of the dual representation of the problem. Thus, if problem 0 could be solved, the organization's problem would be solved. The primary difficulty with problem 0 is that there is a constraint for every extreme point in problem S_K, K=1,...,n. However, by using a clever method these constraints can be generated sequentially. The new feature is that the generation scheme can be interpreted as an iterative exchange of information between the superordinate and each of the subordinates. The information communicated to each subordinate is a vector of goals which contains resource budgets and performance requirements. The information communicated upward from each subordinate is how dissatisfied the subordinate is with the goals which have been set for him, and how this discrepancy dissatisfaction would change if the goals assigned to him change.

The iterative procedure begins with the selection of an initial set of goals, $\underline{\alpha}_K^{\,1}$, for each subordinate $K=1,\ldots,n$. This selection is performed so that

$$\sum_{K=1}^{n} P_K \underline{\alpha}_K \leq \underline{q}.$$

In an established organization the initial selection can be guided by past resource budgets and performance accomplishments.

Using $\underline{\alpha}_K^{\,1}$ subordinate K solves problem S_K to find $z_K(\underline{\alpha}_K^{\,1})$ and $\underline{\pi}_K^{\,1}$. Since $z_K^*(\underline{\alpha}_K^{\,1}) = v_K^*(\underline{\alpha}_K^{\,1})$ at the optimum of problems S_K and D_K, the superordinate solves the following problem to find new goal vectors, $\underline{\alpha}_K^{\,2}$ for $K=1,\ldots,n$:

$$\text{minimize} \qquad \sum_{K=1}^{n} \sigma_K$$

$$\text{subject to:} \qquad \sigma_K \geq z_K^*(\alpha_K^{\,1}) - (\underline{\alpha}_K^{\,1})' \, \underline{\pi}_K^{\,2} + (\underline{\alpha}_K^{\,2})' \underline{\pi}_K^{\,1}$$

$$\sum_{K=1}^{n} P_K \, \underline{\alpha}_K \leq \underline{q}.$$

The entire process can be summarized in the following steps:

(1) The superordinate initially chooses a set of $\underline{\alpha}_K^{\,1}$'s $(K=1,\ldots,n)$ such that

$$\sum_{K=1}^{n} P_K \, \underline{\alpha}_K^{\,1} \leq \underline{q}.$$

He then asks subordinate K: what would you do if you received the vector of goals, $\underline{\alpha}_K^{\,1}$?, i. e., what would you do if you received the resource budgets and the requirements on your performance contained in $\underline{\alpha}_K^{\,1}$?

(2) At stage t of the iterative information exchange process, subordinate K responds by solving the following problem for a fixed $\underline{\alpha}_K^{\,t}$:

$$\text{minimize} \quad z_K(\alpha_K^{\,t}) = \underline{w}_K^{+\prime}\,\underline{d}_K^{\,+} + \underline{w}_K^{-\prime}\,\underline{d}_K^{\,-} + \underline{u}_K^{+\prime}\,\underline{e}_K^{\,+} + \underline{u}_K^{-\prime}\,\underline{e}_K^{\,-}$$

$$\begin{aligned} \text{subject to:} \quad & H_K\underline{x}_K - \underline{d}_K^{\,+} + \underline{d}_K^{\,-} && = \underline{\alpha}_K^{\,t} \\ & G_K\underline{x}_K - \underline{e}_K^{\,+} + \underline{e}_K^{\,-} && = \underline{\beta}_K \\ & A_K\underline{x}_K && \leq \underline{C}_K \\ & \underline{x}_K \geq \underline{0},\ \underline{d}_K^{\,+},\ \underline{d}_K^{\,-},\ \underline{e}_K^{\,+},\ \underline{e}_K^{\,-} \geq \underline{0} \end{aligned} \tag{6-8}$$

He then communicates the optimal solution to this problem, $z_K{*}(\alpha_K^{\,t})$ and $\underline{\pi}_K^{\,*t}$ which is a vector of dual variables associated with the constraints in (6-8), to the superordinate. Note that the j^{th} component of $\underline{\pi}_K^{\,t}$, call it $\pi_{Kj}^{\,t}$, is a measure of how subordinate K's objective function would change if changes were made in the j^{th} component of the goal vector $\underline{\alpha}_K^{\,t}$.

(3) At stage t, the superordinate receives $z_K{*}(\underline{\alpha}_K^{\,t})$ and $\underline{\pi}_K{*}^t$ from each subordinate. He then solves the following problem:

$$\text{minimize} \quad \sum_{K=1}^{n} \sigma_K^{\,t}$$

subject to: $$\sigma_K^{\ t} \geq z_K^*(\underline{\alpha}_K^{\ t}) - (\underline{\pi}_K^{*j})' \ \underline{\alpha}_K^{\ j} + (\underline{\pi}_K^{*j})' \ \underline{\alpha}_K^{t+1} \tag{6-9}$$

for $K=1,\ldots,n$ and

$j=1,\ldots,t$

$$\sum_{K=1}^{n} P_K \underline{\alpha}_K^{t+1} \leq \underline{q} \tag{6-10}$$

The solution to this problem is $\underline{\alpha}_K^{t+1}$ for $K=1,\ldots,n$ which is a new set of goals for the subordinates. Thus, $\underline{\alpha}_K^{t+1}$ is transmitted to subordinate K, and the iterative process of steps 2 and 3 continue until say at iteration p when the solution to the superordinate's problem is $\sigma_K^{\ p^*}$ and $\underline{\alpha}_K^{p+1}$. If the subordinates find a solution such that

$$\sum_{K=1}^{n} z_K^*(\underline{\alpha}_K^{p+1}) = \sum_{K=1}^{n} \sigma_K^{\ p^*}$$

then $\underline{\alpha}_K^{p+1}$ is the optimal goal vector for subordinate K.

Figure 9 depicts the iterative communication process associated with the procedure described above.

The following theorem ensures that the procedure described above converges to an optimal solution.

Theorem 6.4

If one assumes that the set of $\underline{\alpha}_K$ $(K=1,\ldots,n)$ which satisfy $\sum_{K=1}^{n} P_K \ \underline{\alpha}_K \leq \underline{q}$ is closed and bounded, then the iterative procedure described above converges to the optimal solution for the organization's problem in a finite number of iterations.

Superordinate: Sets initial goals for each subordinate.

$\swarrow \underline{\alpha}_k^1$

Stage 1

Subordinate k: Evaluate projects in light of goals set by superordinate and tentatively select project activity levels.

$\swarrow \underline{\pi}_k^1 \quad z_k^*(\underline{\alpha}_k^1)$

Superordinate: Determines new goals for each subordinate.

$\swarrow \underline{\alpha}_k^2$

.
.
.

Stage p

Superordinate: Determines new goals for each subordinate.

$\swarrow \underline{\alpha}_k^{p+1}$

Subordinate k: Evaluates and selects project activity levels.

$\swarrow$

Superordinate: Stop: $\sum_{k=1}^{n} z_k^*(\underline{\alpha}_k^{p+1}) = \sum_{k=1}^{n} \sigma_k^{p*}$; $\underline{\alpha}_k^{p+1}$ is the optimal goal vector for subordinate k.

Figure 9. Iterative Process of Goal Partitioning Procedure.

The proof follows from a number of other works (see Benders [21, p. 245], Lasdon [72, p. 379], Zangwill [125]). This entire procedure has been developed for linear problems; however, the procedure can easily be generalized to the situation where the organization's problem is convex (see Kelley [62] and Zangwill [125]).

Thus, the goal partitioning procedure stated earlier in this chapter avoids the difficulty of having to take convex combinations which is required with Ruefli's "generalized goal" decomposition algorithm.

Lasdon [72, p. 381] discusses the similarity of the Bender's procedure from which the goal partitioning procedure presented earlier was derived and the Dantzig-Wolfe decomposition algorithm. Lasdon concludes that for linear problems, Bender's algorithm and the Dantzig-Wolfe algorithm are really duals of one another.

Some Properties of the Goal Partitioning Procedure

The goal partitioning procedure has several appealing features. At each step of the iterative procedure, the current solution, $\underline{\alpha}_K^t$ $(K=1,\ldots,n)$, satisfies the constraints imposed on the superordinate's behavior. Thus, the partitioning method can be considered as a primal feasible approach, and the iterative information exchange can stop anytime and still ensure that a feasible solution exists for implementation.

As Bender [21, p. 249] and Zangwill [125] point out, an attractive feature of his algorithm (and therefore of the goal partitioning procedure) is the availability of upper and lower bounds on the organization's optimal objective function value. In terms of the organization's problem, a solution is sought which minimizes the sum of the discrepancy dissatisfactions of all the subordinates. The following theorem shows how the superordinate can construct an upper and lower bound on the minimal amount of total dissatisfaction at each iteration of the information exchange process.

Theorem 6.5

Let the optimal objective function value of the organization's problem be given by

$$D = \sum_{K=1}^{n} [\underline{w}_K^{+'} \underline{d}_K^{+*} + \underline{w}_K^{-'} \underline{d}_K^{-*} + \underline{u}_K^{+'} \underline{e}_K^{+*} + \underline{u}_K^{-'} \underline{e}_K^{-*}]$$

At iteration t of the goal partitioning procedure the following inequalities hold:

$$\sum_{K=1}^{n} z_K^*(\alpha_K^t) \geq D \geq \sum_{K=1}^{n} \sigma_K^t \geq \sum_{K=1}^{n} \sigma_K^{t-1} .$$

The proof for this theorem can be found in Benders [21, p. 249] and Lasdon [72, p. 380] and is immediate from duality theory. Thus, the superordinate always knows an upper estimate and a lower estimate for the total minimal dissatisfaction. In addition, the lower bound, $\sum_{K=1}^{n} \sigma_K^t$ is monotonically increasing. Unfortunately, the upper bound $\sum_{K=1}^{n} z_K^*(\alpha_K^t)$, which represents a feasible solution does not monotonically increase. This observation is unfortunate because it means that if the iterative exchange process begins with

$\sum_{K=1}^{n} z_K^*(\alpha_K^1)$ near the optimum, it may take several iterations to decrease the current best upper bound for D.

The difference between the current lower bound and the least upper bound gives the subordinate an idea of how far from optimality the current solution is because the upper and lower bounds will be equal at optimality. This is useful information for two reasons:

(1) If the goal partitioning procedure is terminated before optimality is reached, it gives the superordinate an indication of how far from optimality the solution is.

(2) It has been implicitly assumed that there is no cost (monetary or otherwise) associated with the iterative information exchange in the goal partitioning procedure. In actual practice there would be a cost in terms of money or time. Knowing how much the difference between the upper and lower bounds on the minimal subordinate dissatisfaction decreases from iteration to iteration can facilitate the determination of whether further iterative information exchanges are warranted.

It should be noted that Theorem 6.5 provides information on bounds for the sum of all the subordinate's dissatisfactions, but it does not suggest anything about the bounds on an individual subordinate's dissatisfaction. In fact, a particular subordinate's dissatisfaction may increase or decrease from iteration to iteration.

An Equivalent Formulation for the Superordinate's Problem

It would appear that there is an alternative formulation for the superordinate's problem given by problem O. This alternative is given below:

$$\text{minimize} \quad \sigma^t$$

$$\text{subject to:} \quad \sigma^t \geq \sum_{K=1}^{n} [z_K^*(\alpha_K^{\,t}) - (\underline{\pi}_K^{*j})'\underline{\alpha}_K^j + (\underline{\pi}_K^{*j})'\underline{\alpha}_K^{t+1}]$$

$$\text{for } j=1,\ldots,t$$

$$\sum_{K=1}^{n} P_K \, \underline{\alpha}_K^{t+1} \leq \underline{q}.$$

This alternative form is a surrogated version of (6-9) in the sense that all the inequalities on the subordinates have been summed to give one inequality.[29] Clearly, the above formulation appears to have fewer constraints. However, using the above form might require more iterations to reach optimality than that of (6-9). The trade off between the two formulations would have to be determined from a computational study.

Discussion of the Goal Partitioning Procedure

In this section several important features regarding the economic and behavioral interpretations and implications of the goal partitioning procedure are discussed. These include an exact interpretation of what information is being transmitted and how it is used, an explanation of how the weights associated with goals are determined and affected, and why the goal partitioning procedure can be considered

[29] See Rardin and Unger [88] for some related work.

as a satisficing and compromising technique.

<u>Behavioral Interpretations</u>

The iterative procedure described earlier is envisioned as taking place at discrete time intervals. It is assumed that the organization is complex, and thus the superordinate does not know detailed information about the projects, problems, or constraints facing each subordinate decision making unit. The superordinate is unable to consider all the complexities which enter into the resource allocation decision process [70, p. 545]. Therefore, he concentrates on certain aggregate measures of performance for the organization. From his previous experience, his future expectations, and possibly pressure from factors external to the operating environment such as stockholders, the federal government, labor unions, etc., [35, p. 123] the superordinate forms certain goals. These goals are of three forms:

(1) Goals which relate to the quantities of resources available for use by the organization. For example, he might know the operating budget and the capital expenditure budget. In addition, there may exist only limited supplies of certain skills, equipment, or facilities for the organization's use in carrying out its activities. These kinds of goals are directly related to limited supplies of resources.

(2) Goals which relate to the performance of the organization. Thus, the superordinate sets certain quantifiable (and thus measurable) objectives which he feels the organization should attain. Examples of such goals might be "make a profit of a certain amount," "increase

the company's market share by 2 percent," "reduce major crime by 5 percent," or "decrease traffic fatalities by 10 percent." The assumption of the goal partitioning procedure is that the superordinate can set these objectives. The immediate interest is not with why or how these goals are set other than to hypothesize their existence. Goals of this type are referred to as performance goals.

(3) Goals which relate to how either the resources or performance goals are divided among the subordinate decision units. Examples of this type might include "the police department budget should never exceed the fire department budget," "the libraries budget should be 10 percent of the sum of the department's budgets," and "any money spent on capital improvements should net a 25 percent return."

The task facing the superordinate is how should he partition the goals among the subordinates. The assumption of the goal partitioning method is that the total resource and performance goals should be partitioned and assigned to the subordinate decision units so that the three types of restrictions mentioned above are met, and the total amount of subordinate discrepancy dissatisfaction is minimized. Thus, the three types of goals are reflected in the constraint

$$\sum_{K=1}^{n} P_K \underline{\alpha}_K \leq \underline{q} ,$$

where $\underline{\alpha}_K$ is the vector of goals assigned to subordinate K. It is assumed that these three kinds of goals remain <u>fixed</u> throughout the iterative process.

Subordinate K's behavior is goal directed in the sense that each time a new set of goals is determined for him, he attempts to find a set of project activity levels which do not exceed the technological restrictions common to him, i. e., satisfy $\underline{A}_K \underline{x}_K \leq \underline{C}_K$, and which minimize a weighted deviation from the goals set by the superordinate and by the subordinate. There are several different interpretations that one may associate with a subordinate's weighted deviation. Collomb [33] refers to it as the "internal tension" of subordinate K. The implication is that if the subordinate arrives at a solution which does not deviate from the goals, then there will be no internal pressure within the subordinate's decision unit. On the other hand if the subordinate arrives at a solution which does deviate from the goals established by the superordinate, and the weight associated with these goals is positive, then there is pressure created within the subordinate's decision unit. This pressure can serve as an incentive to generate new alternatives or to reduce the organizational slack [35, p. 36].

A subordinate's weighted deviation also can be interpreted as the unit's discrepancy dissatisfaction. The qualifier "discrepancy" is used to emphasize that any dissatisfaction which results is because of the discrepancy between the goals set either by the superordinate or the subordinate and what the subordinate can accomplish. It is possible for the subordinate to meet the targets set by the superordinate and still be dissatisfied. However, this dissatisfaction is not a result of the deviation (discrepancy).

Clearly, the weights associated with goal deviations have a significant role in the amount of internal tension or discrepancy dissatisfaction. It is explicitly assumed that these weights are determined by the subordinate himself. However, there is no reason to assume that the superordinate cannot set some of them or at least influence their value. One could interpret the amount of a subordinate's autonomy as the power or ability to set the weights by himself. Thus, the more power or authority that a superordinate has in setting or influencing a subordinate's weights, the less autonomy that the subordinate has.

As an example of how the superordinate can influence the value that a subordinate associates with a weight, suppose the superordinate tells the subordinate that if he selects a resource allocation program which requires more resources than he is budgeted for in the goal setting process, then he is fired. This would effect the priority that the subordinate attaches to meeting that goal.

The superordinate's power to influence the subordinate's weights depends upon the type of organization and the reward structure. Although it is assumed that the weights associated with goal deviations are fixed throughout the goal partitioning procedure, it can be generalized so that the weights change during the iterative process, or that the weights are selected by the superordinate or the subordinate from some set. Changing the value of the weights during the partitioning process would have no effect on the convergence properties as long as after some point in time the weights do not change. Allowing the superordinate to directly select all or some of the weights from a set would obviously provide the superordinate with another mechanism for influencing the subordinate's behavior. Note that such a provision would be a direct example of a goal intervention method because it involves the influence of the subordinate's objective function.

Ijiri [59] discusses ways of arriving at numerical values for the weights given that the decision maker can order his priorities for goals. It should be apparent that the weight associated with a goal deviation depends somewhat on the magnitude of the goal level. Thus, the weight may change, depending upon the goal level. However, it seems reasonable to assume that over some domain of goal levels the weights are constant. Therefore, if an organization has some stability, the goal level domain could be easily defined. In a rapidly changing organization, it is much more difficult to specify a domain.

The actual numerical value assigned to a weight depends on several factors. Two important considerations are the reward structure and the subordinate's perception of the superordinate's propensity to change assigned goal levels. Clearly, as the penalty associated with a goal deviation increases, the weight that a subordinate attaches to a deviation should increase. Also, the more receptive the superordinate is to changing goals, the greater the value of the weight should be. This is true because the greater a given subordinate's discrepancy dissatisfaction, the more likely that an adjustment of the goals will decrease the sum of the subordinate's dissatisfactions.

When the superordinate tentatively sets a vector of goals for a subordinate, the subordinate solves his decision problem, problem S_K, finding the resource allocation program he would undertake if in fact these were the goals set for him. In the process he also arrives at how dissatisfied he is with these goals. This discrepancy dissatisfaction is simply the optimal value of his objective function. Since a subordinate's problem is linear, the solution algorithm also gives the values of the dual multipliers associated with the constraints on goals, (6-8). The usual interpretation of the dual multipliers is the change in the value of the objective function per unit change in the right hand side. Thus, the j^{th} component of $\underline{\pi}_K$, is the change in dissatisfaction per unit change in the j^{th} goal.

These two pieces of information, the amount of the subordinate's dissatisfaction, and an indication of how this dissatisfaction would change if the goals set by the superordinate were changed would intuitively be present in an actual allocation process. For example, the subordinate receives a resource budget and certain performance requirements and communicates to the superordinate information about how unreasonable the goals which were set for him are, and then makes an argument for changing them, explaining how much less dissatisfied he would be if they were changed in a certain way.

The superordinate takes this information from each subordinate and uses it to determine if there exists some other partitioning of his goals which might decrease the current amount of subordinate discrepancy dissatisfaction. Specifically, at iteration t of the information exchange the superordinate knows the current best estimate of the total subordinate dissatisfaction. This quantity is given by $\min\limits_{t} \left[\sum_{K=1}^{n} z_K^*(\underline{\alpha}_K^t)\right]$ which is an upper bound on the minimal amount of subordinate dissatisfaction. In the superordinate's problem, $\sum_{K=1}^{n} \sigma_K^t$ is a lower bound on the minimal amount of subordinate dissatisfaction. With the above interpretation for the optimal dual multipliers, $\underline{\pi}_K^*$, one can interpret the constraints in (6-9). Given an $\underline{\alpha}_K^t$, the optimal solution for subordinate K's problem has the property that

$$z_K^*(\alpha_K^t) = \underline{\pi}_K^{*\prime}\underline{\alpha}_K + \underline{\lambda}_K^{*\prime}\underline{\beta}_K - \underline{\mu}_K^{*\prime}\underline{C}_K .$$

Thus, subordinate K's internal dissatisfaction depends on two terms: $\underline{\pi}_K^{*\prime}\underline{\alpha}_K$ and $\underline{\lambda}_K^{*\prime}\underline{\beta}_K - \underline{\mu}_K^{*\prime}\underline{C}_K$. $\underline{\pi}_K^{*\prime}\underline{\alpha}_K$ is discrepancy dissatisfaction which can be attributed to the goals set by the superordinate. $\underline{\lambda}_K^*\underline{\beta}_K - \underline{\mu}_K^*\underline{C}_K$ is discrepancy dissatisfaction which results from the subordinate's goals and "local" constraints. The constraint,

$$\sigma_K^t \geq z_K^*(\underline{\alpha}_K^j) - \underline{\pi}_K^{j\prime}\,\underline{\alpha}_K^j + \underline{\pi}_K^{j\prime}\underline{\alpha}_K^{t+1},$$

states that the superordinate's current estimate on subordinate K's dissatisfaction must be greater than the subordinate's minimal dissatisfaction given goal, $\underline{\alpha}_K^t$, minus the portion of the subordinate's dissatisfaction, $\underline{\alpha}_K^{j\prime}\ \underline{\pi}_K^j$, which is directly attributable to the goals set by the superordinate at iteration j, plus an estimate of the dissatisfaction caused by the new goals which are to be determined at iteration t+1. The reason that several iterations are generally required, i. e., several constraints of the form (6-9) are generated before optimality is reached, is because $\underline{\alpha}_K^{j\prime}\ \underline{\pi}_K^{t+1}$ is only an estimate of dissatisfaction which is good over some range of values for $\underline{\alpha}_K^{t+1}$.

It is claimed that during the iterative information exchange the total discrepancy dissatisfaction of the subordinates will decrease. The superordinate facilitates this decrease by iteratively adjusting the goals $\underline{\beta}_K$. However, the goals determined by the subordinate also affect the amount of discrepancy dissatisfaction. It has been assumed that these goals remain fixed during the process. The amount of discrepancy dissatisfaction caused by deviating from the $\underline{\beta}_K$ goals can change from iteration to iteration, but it is not possible to make any statements about whether it will increase or decrease during the goal partitioning procedure. March and Simon [80] note that if goals are not attainable they may be changed so that they can be reached. Such an assertion would imply that the $\underline{\beta}_K$'s may change through time. The convergence of the goal partitioning procedure is unaffected by these changes so long as after some point in time the $\underline{\beta}_K$'s remain fixed.

To summarize, the goal partitioning approach involves the communication of information between the superordinate and the subordinates. The superordinate is a policy setter in that he sets goals for each subordinate so that the available organization resources are not exceeded, the performance requirements are met, and any requirements on the relationship between the subordinate's resources and performance requirements are satisfied. The information communicated from the subordinates upward, consists of the current amount of discrepancy dissatisfaction or internal tension, and how this dissatisfaction would change if the goals changed. The goal partitioning procedure shows that the information communicated between superordinate and subordinates is at least sufficient in order to find that partitioning of the superordinate's goals which minimizes the total subordinate dissatisfaction.

The concept of goal setting is a form of constraint intervention in the sense that the superordinate passes down goal vectors which serve as the right hand side for constraints in the subordinate's problem. However, it has been pointed out how the superordinate could either directly or indirectly influence the weights in the subordinate's objective function, and thus could also be considered as a goal intervention method. Therefore, one can associate goal partitioning with both constraint and goal intervention methods.

Compromising Final Solutions

Proposition 9

The goal partitioning procedure will likely result in the subordinates arriving at a resource allocation program which does not meet all of the superordinate's goals.

A significant feature of the goal partitioning procedure is that although the superordinate sets the final target levels for resources and performance requirements, the final solution chosen by the subordinate may not meet the overall goals of the superordinate.

To illustrate this feature, suppose $\hat{\underline{x}}_K$ represents the optimal solution to subordinate K's problem given $\hat{\alpha}_K$. Thus, $\hat{\alpha}_K = H_K\hat{\underline{x}}_K - \underline{d}_K^{+} + \underline{d}_K^{-}$. Substituting for $\hat{\alpha}_K$ in (6-10) yields

$$\sum_{K=1}^{n} P_K[H_K\hat{\underline{x}}_K - \underline{d}_K^{+} + \underline{d}_K^{-}] \leq \underline{q} \ .$$

However, there is no assurance that

$$\sum_{K=1}^{n} P_K(H_K\hat{\underline{x}}_K) \leq \underline{q}$$

is satisfied. Therefore, the solution to be implemented by the subordinates may not in fact satisfy the three types of goals mentioned earlier. This can be looked upon in two ways. In the first way, such a result can be envisioned as "enlighted" coordination. That is, the final solution reached is affected by the dissatisfactions of the subordinates. This effect shows that the superordinate is either unwilling or unable to force the subordinates to meet all of the superordinate's goals. It demonstrates that the superordinate is willing to "back off" or compromise on what goals are attained because of his concern over the subordinates' dissatisfaction.

The second way of looking at this feature is that it is undesirable because there may be certain goals which the superordinate cannot compromise. In this case the superordinate must create a reward structure which ensures that if it is technologically possible, certain goals are met. This would involve influencing the subordinate's weights attached to deviations from goals.

Satisficing Behavior

Ruefli [90, p. 28] argues that since the subordinate's problem in his goal decomposition approach has a goal programming formulation, then each subordinate is really not optimizing but is in fact "satisficing." If this statement is true, then of course it could be applied to the goal partitioning procedure presented in this chapter. However, this author would challenge the statement that goal programming is a satisficing approach in the sense of Simon, who first used the term. To be satisficing, March and Simon [80, p. 140] state that there must exist a set of criteria that describes minimally satisfactory alternatives, and a selected alternative must meet or exceed all these criteria. But, goal programming seeks to find a set of alternatives which minimizes some weighted deviation from certain goals. Therefore, to make a goal programming formulation represent satisficing, it is necessary to identify the satisficing region and then make the weights for deviating from this region very high.

A slightly generalized concept of satisficing can be represented through goal interval programming [33, 42]. The basic idea of the approach is that for any goal, there is some interval of performance in which the decision maker is completely satisfied. As performance moves away from this interval, the decision maker becomes increasingly dissatisfied.

In order to express such a scheme consider the following "amended" decision problem for subordinate K.

Problem S_K'

$$\text{minimize} \quad \sum_{j=1}^{r} [\underline{w}_{Kj}^{+'} \underline{d}_{Kj}^{-'} + \underline{w}_{Kj}^{-'} \underline{d}_{Kj}^{-} + \underline{u}_{Kj}^{-'} \underline{e}_{Kj}^{'} + \underline{u}_{Kj}^{-'} \underline{e}_{Kj}]$$

$$\text{subject to:} \quad H_K\underline{x}_K + \sum_{j=1}^{r} [-\underline{d}_{Kj}^{+} + \underline{d}_{Kj}^{-}] = \underline{\alpha}_K$$

$$G_K\underline{x}_K + \sum_{j=1}^{r} [-\underline{e}_{Kj}^{+} + \underline{e}_{Kj}^{-}] = \underline{\beta}_K$$

$$A_K\underline{x}_K \leq \underline{C}_K$$

$$\left.\begin{array}{l} \underline{0} \leq \underline{d}_{Kj}^{+} \leq \underline{l}_{j+1}^{+} - \underline{l}_{j}^{+} \\ \underline{0} \leq \underline{d}_{Kj}^{-} \leq \underline{l}_{j+1}^{-} - \underline{l}_{j}^{-} \\ \underline{0} \leq \underline{e}_{Kj}^{+} \leq \underline{v}_{j+1}^{+} - \underline{v}_{j}^{+} \\ \underline{0} \leq \underline{e}_{Kj}^{-} \leq \underline{v}_{j+1}^{-} - \underline{v}_{j}^{-} \end{array}\right. \quad j=1,\ldots,r$$

$$\underline{x}_K \geq \underline{0}, \quad \underline{d}_{Kj}^{+}, \quad \underline{d}_{Kj}^{-}, \quad \underline{e}_{Kj}^{+}, \underline{e}_{Kj}^{-} \geq \underline{0},$$

where $\underline{d}_{Kj}^{+}$ is a vector of deviations in the j^{th} interval and $[l_{j+1}^{+}, l_{j}^{+}]$ defines the j^{th} interval for a positive deviation from the goals set by the superordinate. To demonstrate this approach, suppose the superordinate desires a profit level of between 6 and 8 units. If profit exceeds 8, the chances of government action increases, and if profit falls below 6, the stockholders will get aroused. A profit level outside this interval may be acceptable but is to be avoided if possible. In addition, as the profit level moves away from this interval the dissatisfaction increases in a manner given by Figure 10.

From Figure 10 the dissatisfaction could be written as:

$$0d^{+1} + 1/2\ d^{+2} + d^{+3} + 0d^{-1} + d^{-2} + 3/2d^{-3} + 4d^{-4}$$

where
$$0 \leq d^{+1} \leq 8-7 = 1$$
$$0 \leq d^{+2} \leq 10-8 = 2$$
$$0 \leq d^{+3}$$
$$0 \leq d^{-1} \leq 7-6 = 1$$
$$0 \leq d^{-2} \leq 6-5 = 1$$
$$0 \leq d^{-3} \leq 5-3 = 2$$
$$0 \leq d^{-4}$$

The constraint concerned with profit would be

$$px_1 + p_2x_2 + \ldots + p_mx_m - d^{+1} - d^{+2} - d^{+3} + d^{-1} + d^{-2} + d^{-3} + d^{-4} = 7$$

Thus, the proposed model can easily be altered to reflect a satisficing type of behavior. The only restriction that must be made is that the dissatisfaction function be convex. This restriction is necessary to maintain the subordinate's problem as a convex programming problem. Computationally and in terms of interpretation, this approach creates no difficulty; however, the notation and mathematical representation is quite cumbersome. Therefore, the form of Problem S_K' will not be written, but instead the less complex form will be used where the piecewise approach is not used.

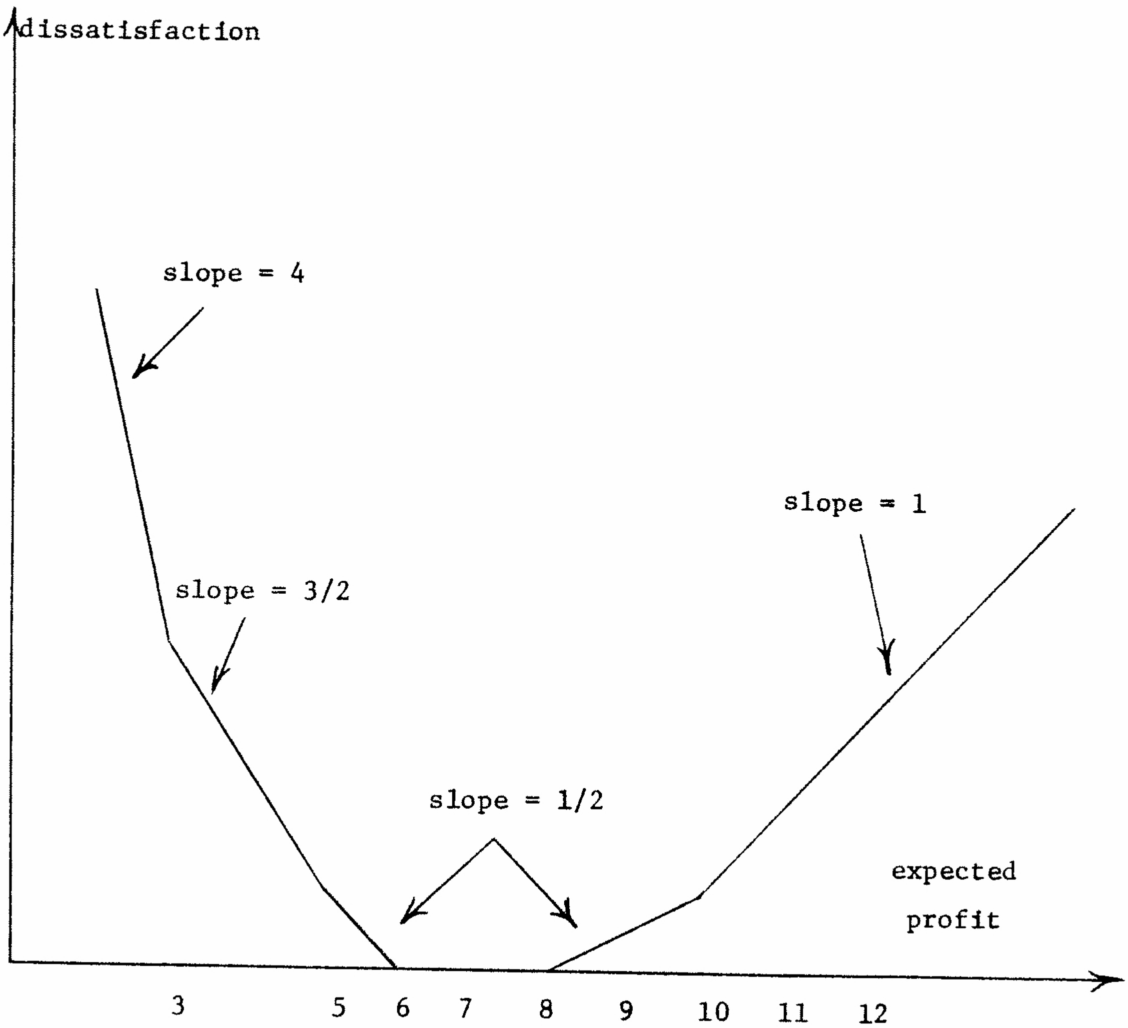

Figure 10. Example of Dissatisfaction Function.

The goal partitioning procedure displays the characteristics of bounded rationality and can be considered as a generalized satisficing approach. According to Simon [103] for the organization to find an optimal set of project activity levels to undertake, it is necessary to have an exhaustive set of criteria which considers all factors that are influenced by different alternatives. The rational decision maker then optimizes with respect to these criteria. However, because of man's bounded rationality, he makes his choices based upon "a simple picture of the situation that takes into account just a few of the factors that he regards as most relevant and crucial" [104, p. XXVI]. This is exactly what the superordinate does when he selects his goals which are represented by $\underline{q}$ in the constraint

$$\sum_{K=1}^{n} P_K \underline{\alpha}_K \leq \underline{q}.$$

Instead of trying to construct some overall grandiose utility function, the superordinate settles for a few measures of effectiveness, e. g., profit, market share, liquidity position, etc. In this sense the goal partitioning procedure recognizes the "bounded rationality" of man.

Summary of Behavioral Aspects

The goal partitioning procedure presented earlier in this chapter has several appealing aspects. One of the more important features is summarized in proposition 10.

Proposition 10

In a non-cooperative organization a goal partitioning procedure may lead to a resource allocation program which is different from that which would be selected by the superordinate or the subordinate acting in isolation, but it will be a program which reflects both the goals of the superordinate and the goals of the subordinates.

Both the superordinate and the subordinates influence the resource allocation program selected by the organization. The superordinate is a policy setting entity. He sets goals for each of the subordinates in such a way as to ensure that the resource supply is not exceeded,to ensure that certain measures of performance are met, and so that any other restrictions concerning the way resources or performance requirements are assigned to subordinates are met. The concept of goal setting for each subordinate is in some sense related to aspiration level theory, e. g., see Frank [45], Chapman and Volkman [26], Atkinson [11] and Charnes and Stedry [31]. Probably the study most relevant to the goal setting displayed in the goal partitioning procedure is that of Stedry [110]. He used experiments to develop a mathematical model which showed that performance is better when the goals set by an external source are slightly higher (possibly even unattainable) than the subject's aspiration level. However, if the goals are set much higher than what can be attained, then performance is very poor. Thus, if the superordinate sets goals which are somewhat above what a subordinate can attain, the subordinate may perform better. The use of a goal programming formulation alleviates the mathematical difficulties associated with setting goals for which no feasible solution exists.

The information which is exchanged by the superordinate and the subordinates allows the organization to arrive at a set of project activity levels in a finite number of information exchanges. The procedure recognizes the "bounded rationality" of the organization, and it also explicitly allows for the superordinate to compromise on what goals are attained because of his concern over the discrepancy dissatisfaction of the subordinates.

In the remaining two sections of this chapter two different types of negotiation models are presented. One form recognizes that the organization might be interested in minimizing the maximum dissatisfaction of any one subordinate rather than maximizing their total dissatisfaction. The second form, which has been proposed by Ruefli [92] and Collomb [33], allows for the superordinate to change the solution found because of his own dissatisfaction.

Minimizing the Maximum Subordinate Dissatisfaction

It seems reasonable that instead of the superordinate desiring to minimize the sum of the subordinates dissatisfaction, the superordinate might be more concerned with keeping each subordinate's dissatisfaction as small as possible. In this section it is shown how such an objective can be handled mathematically via a partitioning procedure. Essentially it is shown that with the above objective the overall problem can still be stated in a linear form, and then Bender's partitioning can be applied.

In the case at hand the overall organization's problem can be stated as:[30]

[30] The same notation and interpretation is the terms as before.

$$\text{minimize} \quad \underset{K=1,\ldots,n}{\text{maximum}} \left[\underline{w}_K^{+'} \underline{d}_K^{+} + \underline{w}_K^{-'} \underline{d}_K^{-} + \underline{u}_K^{+'} \underline{e}_K^{+} + \underline{u}_K^{-'} \underline{e}_K^{-}\right]$$

$$\begin{array}{lll}
\text{subject to:} & H_K\underline{x}_K - \underline{d}_K^{+} + \underline{d}_K^{+} - \underline{\alpha}_K & = \underline{0} \\
& G_K\underline{x}_K - \underline{e}_K^{+} + \underline{e}_K^{-} & = \underline{\beta}_K \\
& A_K\underline{x}_K & \leq \underline{C}_K \\
& \sum_{K=1}^{n} P_K\underline{\alpha}_K & \leq \underline{q} \\
& \underline{x}_K \geq \underline{0},\ \underline{d}_K^{+},\ \underline{d}_K^{-} \geq \underline{0}. &
\end{array}$$

As stated above the problem is nonlinear, but it can be given in an equivalent form which is linear. This formulation is given below:

$$\begin{array}{lll}
\text{minimize} & z & \\
\text{subject to:} & z \geq \underline{w}_K^{+'} \underline{d}_K^{+} + \underline{w}_K^{-'} \underline{d}_K^{-} + \underline{u}_K^{+'} \underline{e}_K^{+} + \underline{u}_K^{-'} \underline{e}_K^{-} & \\
& H_K\underline{x}_K - \underline{d}_K^{+} + \underline{d}_K^{-} - \underline{\alpha}_K & = \underline{0} \\
& G_K\underline{x}_K - \underline{e}_K^{+} + \underline{e}_K^{-} & = \underline{\beta}_K \\
& A_K\underline{x}_K & \leq \underline{C}_K \\
& \sum_{K=1}^{n} P_K\underline{\alpha}_K & \leq \underline{q} \\
& \underline{x}_K \geq \underline{0};\ \underline{d}_K^{+},\ \underline{d}_K^{-},\ \underline{e}_K^{+},\ \underline{e}_K^{-} \geq \underline{0} &
\end{array}$$

It is possible to break this problem up and apply a procedure exactly like the goal partitioning process given on pages 161 to 163 with the following exception. At iteration t the superordinate chooses new goal vectors, $\underline{\alpha}_K^{t+1}$, by solving the following problem:

$$\text{minimize } z^t$$

$$\text{subject to: } z^t \geq z_K^*(\underline{\alpha}_K^{t}) - (\underline{\pi}_K^{*j})' \, \underline{\alpha}_K^{j} + (\underline{\pi}_K^{j})' \, \underline{\alpha}_K^{t+1}$$

$$\text{for } K=1,\ldots,n \text{ and}$$

$$k=1,\ldots,t$$

$$\sum_{K=1}^{n} P_K \underline{\alpha}_K^{t+1} \leq \underline{q}$$

The iterative process continues until at some iteration p when the solution to the superordinate's problem is z^{p*} and $\underline{\alpha}_K^{p+1}$. If, given $\underline{\alpha}_K^{p+1}$, the subordinates find a solution such that

$$\underset{K=1,\ldots,n}{\text{minimum}} \quad z_K^*(\underline{\alpha}_K^{p+1}) = z^{p*},$$

then $\underline{\alpha}_K^{p+1}$ is the optimal goal vector for subordinate K.

The assertions that this scheme converges finitely and that there exist upper and lower bounds on the optimal value of the objective function can be shown using Bender's results [21]. The question of whether the formulation where the superordinate tries to minimize the maximum dissatisfaction is a better description than the superordinate manimizing the total dissatisfaction is unanswered. Empirical studies are needed to resolve the issue. The purpose here is only to illustrate that alternative forms do exist and can be handled mathematically. For a particular organization it would be interesting to determine how the solutions compare.

A Negotiation Model Which Considers the Dissatisfaction of Both the Superordinate and the Subordinates

Returning to the mathematical model attributed to Ruefli in the beginning of the chapter, recall that the objective of the overall problem was to

$$\text{minimize} \quad \sum_{K=1}^{n} [\underline{w}_K^{+'} \underline{d}_K^{+} + \underline{w}_K^{-'} \underline{d}_K^{-}]$$

which can be interpreted as the minimization of the sum of the subordinates dissatisfaction. Collomb [33, p. 121] has suggested that such an objective would probably be representative of a government agency but not a progressive competitive organization. Two recent studies [33, 92] have relaxed this assumption.

Collomb [33] in his Ph.D. dissertation states the organization's problem so that a solution is sought which minimizes the subordinates' weighted deviations from the goals and the superordinate's weighted deviation from his goals. Specifically, the superordinate finds $\underline{\alpha}_K$, the goals for subordinate K, by solving the following problem:

$$\text{minimize} \quad \underline{w}^{+'} \underline{d}^{+} + \underline{w}^{-'} \underline{d}^{-}$$

$$\text{subject to:} \quad \sum_{K=1}^{n} P_K[\underline{\alpha}_K + \underline{d}_K^{+} - \underline{d}_K^{-}] - \underline{d}^{+} + \underline{d}^{-} = \underline{G}_0$$

$$\underline{d}^+, \underline{d}^- \geq \underline{0}$$

where $\underline{d}^+$, $\underline{d}^-$ are m_o x 1 vectors of deviations, and

$\underline{w}^+$, $\underline{w}^-$ are m_o x 1 vectors of weights associated with the deviations.

The other terms are defined as before. For a fixed $\underline{\alpha}_K$ subordinate K solved his problem which is given by:

$$\text{minimize} \quad \underline{w}_K^+ \underline{d}_K^+ + \underline{w}_K^- \underline{d}_K^-$$

$$\text{subject to:} \quad H_K \underline{x}_K - \underline{d}_K^+ + \underline{d}_K^- = \underline{\alpha}_K$$

$$\underline{x}_K \geq \underline{0}, \ \underline{d}_K^+, \ \underline{d}_K^- \geq \underline{0}.$$

Collomb shows [32, p. 9] how to write these problems as the following overall problem:

<u>Collomb's Problem</u>

$$\text{minimize} \quad \underline{w}^{+\prime} \underline{d}^+ + \underline{w}^{-\prime} \underline{d}^- \tag{6-11}$$

$$\text{subject to:} \quad \sum_{K=1}^{n} [P_K(\underline{\alpha}_K + \underline{d}_K^+ - \underline{d}_K^-] - \underline{d}^+ + \underline{d}^- = \underline{G}_o \tag{6-12}$$

$$H_K \underline{x}_K - \underline{d}_K^+ + \underline{d}_K^- = \underline{\alpha}_K \tag{6-13}$$

$$\begin{aligned} \underline{\pi}_K' H_K &\leq \underline{0} \\ -\underline{\pi}_K &\leq \underline{w}_K^+ \\ \underline{\pi}_K &\leq \underline{w}_K^- \end{aligned}$$

$$\underline{w}_K^{+}\underline{d}_K^{+} + \underline{w}_K^{-}\underline{d}_K^{-} \leq \underline{\pi}_K{}'\underline{\alpha}_K \tag{6-14}$$

$$\underline{d}_K^{+}, \underline{d}_K^{-} \geq \underline{0}, \underline{d}^{+}, \underline{d}^{-} \geq \underline{0}$$

Next, he points out [32, p. 146] that this problem is non-convex and nonlinear ($\underline{\pi}_K$ is a variable) and then applies an appropriate algorithm to solve the problem. He concludes that any ordinary approximation process by which the superordinate would proceed by incremental changes in accordance with the deviations observed is most likely to lead to a local suboptimum.

After a careful analysis the following observations can be made regarding Collomb's approach:

(1) The superordinate does explicitly consider the possibility that the subordinate may not attain the goals set for him through relationship (6-12).

(2) Although it is not apparent, the overall objective of Collomb's problem is to find goal levels, $\underline{\alpha}_K$, K=1,...,n, and activity levels $\underline{x}_K$, K=1,...,n, so that the sum of the superordinate's weighted deviations and the subordinate's weighted deviations is minimized. This is true because the objective function, (6-11), is to minimize the weighted deviations from the superordinate's goals and the constraints in (6-13) are simply the constraints for the primal and dual of the subordinates' problems. Constraint (6-14) states that the value of the dual objective function must be greater than or equal to the value of the primal objective function for each subordinate's problem. However, from duality theory it is known that for any feasible primal

and dual solution, i. e., one which satisfies (6-13), that

$$\underline{w}_K^{+'}\underline{d}_K^{+} + \underline{w}_K^{-'}\underline{d}_K^{-} \geq \underline{\pi}_K'\underline{\alpha}_K ,$$

therefore constraint (6-14) coupled with the constraints in (6-13) implies that

$$\underline{w}_K^{+'}\underline{d}_K^{+} + \underline{w}_K^{-'}\underline{d}_K^{-} = \underline{\pi}_K'\underline{\alpha}_K,$$

which is the optimality condition for each subordinate's problem. Thus, Collomb is assuming that the overall objective function is to minimize the sum of the superordinate's dissatisfaction and the subordinate's dissatisfaction, and these objectives are equally weighted. The superordinate's dissatisfaction (a term used by this author) is given by the weighted deviation from the superordinate's goals, i. e., $\underline{w}^{+'}\underline{d}^{+} + \underline{w}^{-'}\underline{d}^{-}$ is the superordinate's dissatisfaction.

Ruefli [92] has made two extensions to Collomb's work. First, he allows the superordinate to trade off the two main components of dissatisfaction, i. e., the superordinate's dissatisfaction and the subordinate's total dissatisfaction. Second, he avoids the difficulties incurred by Collomb by suggesting the use of generalized linear programming as a solution procedure. Specifically, using Ruefli's approach the overall problem facing the organization is

$$\text{minimize} \quad \underline{w}^{+'}\underline{d}^{+} + \underline{w}^{-'}\underline{d}^{-} + \hat{\underline{w}}'\left(\sum_{K=1}^{n} [\underline{w}_K^{+'}\underline{d}_K^{+} + \underline{w}_K^{-'}\underline{d}_K^{-'}]\right)$$

subject to: $$\sum_{K=1}^{n} P_K[H_K\underline{x}_K - \underline{d}_K^+ + \underline{d}_K^-] - \underline{d}^+ + \underline{d}^- = \underline{G}_o$$

$$H_K\underline{x}_K - \underline{d}_K^+ + \underline{d}_K^- = \underline{\alpha}_K \quad \text{for } K=1,\ldots,n$$

$$\underline{x}_K \geq \underline{0},\ \underline{d}_K^+,\ \underline{d}_K^- \geq \underline{0},\ \underline{d}^+,\ \underline{d}^- \geq \underline{0}$$

Thus, by studying the overall problem in the above form shows that the superordinate is weighting the importance of the subordinate's dissatisfaction and his own dissatisfaction. Ruefli proposes that this problem can be solved in a decentralized fashion using generalized linear programming. However, the usual problem of choosing convex combinations is still present. However, this problem could be overcome by using a partitioning scheme.

To summarize, both Collomb and Ruefli have attempted to relax the assumption that the superordinate seeks to partition out his goals so as minimize only the subordinates' discrepancy dissatisfactions. Ruefli's extension is more general because he allows the superordinate to trade off his dissatisfaction with that of the subordinates.

Summary

The objective in this chapter has been to investigate the use of negotiation models for conceptualizing the coordination of resource allocation decisions in a two level hierarchical organization where there is conflict between the superordinate and the subordinates over what objectives ought to be pursued. The primary contribution is the presentation and analysis of a goal partitioning procedure based on the earlier work of Benders [21], Kelley [62], Dantzig [36], Zangwill [125] and Ruefli [91]. This procedure explicitly recognizes the conflict over objectives as well as the competition among subordinates for limited resources. These conflicts are resolved via a bargaining process in which both the superordinate and the subordinates participate. The goal partitioning procedure resembles a constraint intervention mechanism, but it also can be shown to relate to a goal intervention mechanism.

The chapter began with a description of Ruefli's work. It was shown how the use of generalized linear programming as a representation of the solution procedure assumes that the subordinates solve their problem as a group, and that the subordinates may select any combination of the goal levels which are assigned to them during the iterative process. Ruefli's concepts were then extended by considering a slightly more general model, and by using a partitioning procedure derived from previous work of Kelley, Benders, Dantzig, and Zangwill. The properties of the goal partitioning method were discussed. The information which is communicated during the exchange process was analyzed, and the economic and behavioral interpretations and implications associated with the mathematical representation of both the superordinate's and the subordinates' decision problems were given. Finally, two alternative objectives for the coordination process were presented.

The negotiation models discussed in this chapter exemplify Simon's concept of bounded rationality. That is, instead of trying to optimize some overall grandiose organizational utility function, a resource allocation program is sought which gets "as close as possible" to meeting some goals which are fixed by the superordinate, the subordinate himself, and by external factors. How close the organization comes to accomplishing these goals is influenced by the weights associated with the deviations by subordinates. There are several different interpretations that one may associate with this weighted deviation such as discrepancy dissatisfaction and internal tension.

The goal partitioning procedure which is described in this chapter is an iterative scheme whereby the superordinate and the subordinates exchange information. However, both the superordinate and the subordinates maintain their informational autonomy in the sense that neither unit must communicate information about its objective function or its constraint set. The iterative information exchange procedure converges in a finite number of exchanges to a solution which minimizes the sum of the weighted deviations of the subordinates from goals set by both the superordinate and the subordinate himself. The final program reached does depend on the structure of the organization. The final solution arrived at is not necessarily the one which the superordinate might have selected had he made all the allocation decisions himself. Thus, the goal partitioning procedure is a relative coordination mechanism.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

The primary focus of this research study has been on the development, interpretation, and analysis of analytical models for conceptualizing the resource allocation decision process in a hierarchical decentralized organization. The three main classes of results are:

(1) The integration under a common framework (from the viewpoint of someone interested in structuring information flow in organizations) of existing mathematical techniques for characterizing the resource allocation decision process in a hierarchical decentralized organization.

(2) Using the theory of mathematical programming as a tool for analysis a number of propositions regarding behavior in organizations were stated. These could serve as the basis for future empirical studies.

(3) The development of negotiation models for coordinating resource allocation decisions when there exists conflict between levels over objectives.

The integrating framework recognizes that there are two potential sources of conflict within a hierarchical decentralized organizations. There may exist conflict between subordinate decision making units over sharing limited resources. An organization where this is the only source of conflict is referred to as a "cooperative organization." On the other hand, the breaking up of the organization into several different decision making entities can cause conflict between levels in the hierarchy over what objectives should be pursued. An organization which has both kinds of conflict is referred to as a "non-cooperative organization."

To overcome these conflicts there exists two main classes of coordination mechanisms: coordination through goal intervention and coordination through constraint intervention. Although both classes of mechanisms have the same motive, viz., to influence the subordinate decision making units to undertake a resource allocation plan which does not exceed the available resources and which furthers the objectives of the superordinate, there are some significant differences in interpretation and application.

The analysis portion of the study was divided into two sections: the study of goal and constraint intervention coordination mechanisms in a cooperative organization and the study of negotiation models for coordination in a non-cooperative organization. While most of the results in Chapters III and IV are not novel, e. g., see the mathematical programming literature [46, 47, 54, 72], they have never been synthesized in the framework of the resource allocation decision process in a hierarchical decentralized organization. The models presented for the analysis of goal and constraint intervention coordination mechanisms in cooperative organizations were normative, i. e., they were concerned with how decision making units ought to behave. However, in many instances these models are descriptive of typical resource allocation coordination processes. Chapters III and IV investigated the adequacy and appropriateness of the existing models as descriptions of the coordination process in typical organizations.

When goal and constraint intervention techniques are applied in a cooperative organization, they are often called pricing and resource budgeting techniques. The primary motivation for most existing analytical models of pricing and budgeting has come from mathematical decomposition theory. In Chapters III and IV, the pricing and budgeting mechanisms were derived from the general mathematical model of resource allocation decision making suggested in Chapter II. In Proposition 2 it was claimed that resource budgeting techniques are preferable to pricing techniques for coordinating in a cooperative organization because they work under more general conditions and the solutions are always feasible. Proposition 1 suggested the conditions under which pricing methods could not be used to find the overall optimal solution.

The analysis of resource budgeting methods in Chapter IV suggested that if management allocates resources based on information about just one alternative, e. g., its allocation last period, then it is quite unlikely that the overall optimal solution can be found. This seems particularly relevant since many organizations perform the budgeting activity with prime consideration being a subordinate's budget last period.

Chapter V described some of the behavioral implications inherent in the coordination mechanisms for cooperative organizations. Specifically, it was shown that:

(1) The only conflict is a result of the subordinates competing for limited organizational resources.

(2) The structure of the organization has no effect on the final solution reached (Proposition 4).

(3) The subordinate decision making units have no autonomy.

(4) The allocation decision reflects only the objectives of the superordinate.

Chapter V also investigated the result when a pricing or resource budgeting procedure is used to coordinate in a non-cooperative organization. Such an investigation is relevant because it is quite conceivable that in a decentralized organization the superordinate might believe that his subordinates have the same utility function that he does, i. e., the superordinate might believe the organization is cooperative. Propositions 5 to 8 indicate that neither pricing mechanisms nor budgeting procedures are suitable for coordinating in non-cooperative organizations. An example is used to illustrate how the use of coordination mechanisms which work in cooperative organizations may result in a poor solution being found with respect to both the superordinate's and the subordinates' utility functions.

In Chapter V it was also shown that even if the superordinate has complete information about the subordinate's constraint sets, it may be impossible to use a pricing scheme to influence the subordinate to find the superordinate's optimum. These results indicate that neither pricing nor budgeting approaches are satisfactory coordination mechanisms in a non-cooperative organization.

In Chapter VI a class of coordination mechanisms called negotiation models which allow the resource allocation decision to be influenced by both the superordinate and the subordinates was introduced. The important aspects of these models are:

(1) They allow for informational autonomy on the part of both the superordinate and the subordinate.

(2) The structure of the organization can affect the final decision. In other words, the final resource allocation program selected may be different under different organization structures. For example, the decisions made under a centralized structure may differ from those made under a decentralized structure.

(3) The goal setting behavior of the superordinate and the subordinates explicitly takes into account the "bounded rationality" of man, i. e., the decision makers concentrate only on certain aggregate measures of performance rather than trying to optimize some grandiose utility function.

(4) The models explicitly allow for both the superordinate and the subordinates to have their own set of goals.

(5) In the case of the goal partitioning procedure the iterative information exchange process between superordinate and subordinates converges in a finite number of information exchanges to a solution which minimizes the total weighted deviation from the goals set by the superordinate and the subordinates.

(6) The goal partitioning procedure may lead to a resource allocation plan being selected which is different from one which would be selected by the superordinate or the subordinates acting in isolation; however, it will lead to a program which reflects both the goals of the superordinate and the goals of the subordinates.

It is felt that the concepts used in this research have wide applicability: in industry, government, profit seeking enterprises, etc. While the analytical models discussed throughout the dissertation are theoretical, they do lend validity to the use of certain kinds of coordination mechanisms in certain environments. Hopefully, the framework used in this research provides a basis by which various coordination mechanisms can be contrasted and compared. Finally, the results suggest a starting point for empirical studies.

Recommendations for Further Study

During the course of this research study several areas for future investigation were uncovered. These can be grouped into three classifications: empirical, mathematical, and supplementary. The extensions in the empirical area include:

(1) For different kinds of organizations test the propositions which were stated in Chapters III to VI.

(2) Which objective is more appropriate as a description of goal seeking behavior: minimizing the sum of weighted deviations for all subordinates or minimizing the maximum weighted deviation for the subordinates. Possibly, a more important question to investigate is: how do the solutions differ under each of the two criteria?

(3) Investigate whether there are any organizations within which quantifiable measures exist which correspond to the information which is assumed to be communicated during the iterative procedure associated with the goal partitioning method.

(4) In a typical organization how many iterative exchanges of

information is normal before a resource allocation program is settled on for implementation.

(5) The goal partitioning procedure suggests that no one participant dominates the resource allocation decision process. Some empirical studies substantiate this, e. g., see Baker, et al. [14], Connolly [34], and Shumway, et al. [101]. Additional studies should investigate this hypothesis more fully.

(6) The possibility of running the goal partitioning procedure as a shadow model could be investigated, i. e., while tha actual decision process is going on, the partitioning procedure could also be utilized. This might indicate discrepancies in the actual planning process or in the partitioning procedure.

(7) In a sense the use of coordination mechanisms can cause an organization to become more centralized. The differences between centralization acts and coordination acts should be investigated. The author conjectures that coordination mechanisms are really centralizing processes even though they allow for decentralized decision making. It seems important to analyze the long term effect of different coordination mechanisms on the degree of centraliaation.

The recommendations for further study in the area of mathematical analysis are:

(1) Investigate ways to consider communications and dependencies (externalities) between subordinate decision making units.

(2) As Geoffrion [46] noted there are many unanswered questions regarding the computational efficiency of different algorithms. There is a definite need for extensive testing of various algorithms for

describing the iterative decision process in a hierarchical decentralized organization.

(3) The computational efficiency of the goal partitioning procedure should be investigated. Ways in which the procedure could be made more efficient such as dropping some of the non-binding constraints in the superordinate's problem (see Geoffrion [46, p. 382]) should be studied. The question of how quickly the goal partitioning procedure converges to a "near" optimum should be answered.

(4) The goal partitioning procedure could be used to investigate the effect of interdependencies between time periods and the effect of incremental increases in the superordinate's performance targets through time.

Supplementary research could study measurement problems. For example, the problem of how to determine the goals of the superordinate and the subordinates, and how to assign weights to deviations from goal levels, should be investigated. In addition, the relationships between the negotiation models discussed in Chapter VI and the theory of teams [81] should be investigated. Also, the basic concepts of the negotiation models could be used to study non-hierarchical organizations such as funder-user-servicer type organizations [13].

The possibility of developing a metric for gauging the centralization-decentralization properties of different types of organizations and organizational structures should be pursued. In Chapter V it was shown how the superordinate and the subordinates may have different optimal solutions. That discussion might provide the foundation for the metric.

One of the pruposes of this research has been to use analytical models to conceptualize the resource allocation decision process in hierarchical decentralized organizations. Most of the models discussed in the dissertation have limited usefulness from an operational viewpoint. There is a need for the development of mathematical procedures which recognize the inabilities of decision makers to specify multi-criterion utility functions. Recently, Geoffrion [49] and Geoffrion and Hogan [50] have presented interactive models which do not require the explicit statement of utility functions. Their work should be extended to a hierarchical decentralized organization where there are several decision making units.

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